Lecture Notes · Unit 8 · Lecture 12 AP CED 8.12

Washers, other axes

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO: AN AXIS OUTSIDE THE REGION

Washer Method: other Axes AP Calculus 8.12

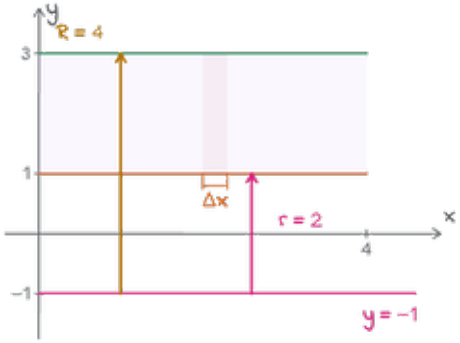
Let's get that 5

Axis outside → R, r from where?

IDEA 1: BOTH RADII FROM THE AXIS

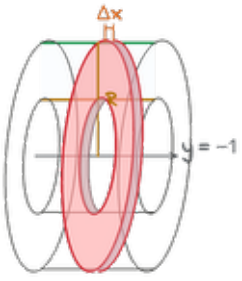
Both radii from the axis

$1 \le y \le 3$, $y = -1$ → from the axis $R = 4$ $r = 2$



1. Both radii from the axis

$R = 4, r = 2 \rightarrow V = \pi \int_0^4 (16 - 4)\,dx = 48\pi$



Hollow pipe

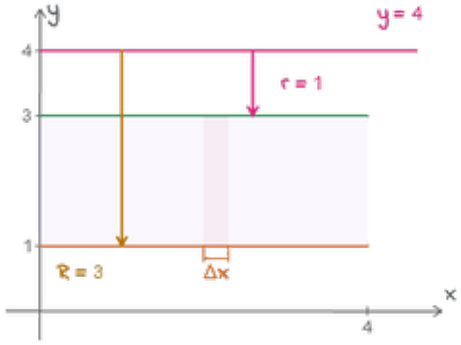
Axis below: $y = k$ R = far $-k$ r = near $-k$

IDEA 2: AN AXIS ABOVE

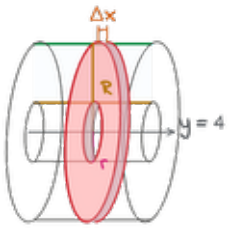
Axis above: R and r trade places

2. An axis above

Same band, around $y = 4$ → far: $R = 4 - 1 = 3$, near: $r = 4 - 3 = 1$



Washer → $V = \pi \int_0^4 (9 - 1)\,dx = 32\pi$ → new solid



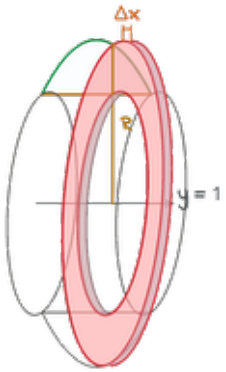
Axis $x = k$ → same, in y

AP → washers around a line like $y = -1$

PROBLEM 1 OF 2

Around y = 1

S is the region between $y = 4 - x^2$ and $y = 3$. Find the volume when S is revolved around $y = 1$.



Solution

R = far curve - 1, r = near curve - 1; pi times the integral of R^2 - r^2

Plan: ends → R, r from y = 1 →

$$\pi\int (R^2 - r^2)\,dx$$

$$4 - x^2 = 3 \rightarrow x = \pm 1$$

$$R = (4 - x^2) - 1 = 3 - x^2 \rightarrow r = 3 - 1 = 2$$

$$V = \pi\int_{-1}^{1} \left((3 - x^2)^2 - 2^2\right)dx$$

$$= \pi\int_{-1}^{1} (5 - 6x^2 + x^4)\,dx$$

$$= \pi\left(10 - 4 + \frac{2}{5}\right) = \frac{32\pi}{5}$$

Workbook → mark the axis first

Careful → a common mistake

X Common mistake

$$4 - x^2, 3 \rightarrow \frac{136\pi}{15}$$

✓ Do this

$$R = 3 - x^2, r = 2 \rightarrow \frac{32\pi}{5}$$

PROBLEM 2 OF 2

Around x = -1

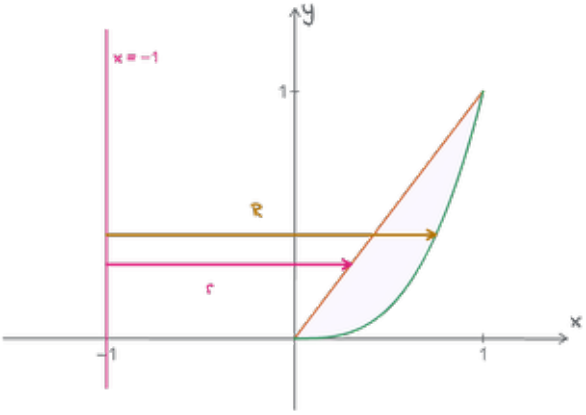
The region between $y = x$ and $y = x^3$ in the first quadrant is revolved around the line $x = -1$. Which integral gives the volume?

(A) $\pi\int_0^1 \left((\sqrt[3]{y} + 1)^2 - (y + 1)^2\right)dy$

(B) $\pi\int_0^1 \left((y + 1)^2 - (\sqrt[3]{y} + 1)^2\right)dy$

(C) $\pi\int_0^1 (\sqrt[3]{y} - y)^2\,dy$

(D) $\pi\int_0^1 \left(y^{\frac{2}{3}} - y^2\right)dy$



Solution

A vertical axis: dy; both radii from x = -1

Plan: axis x = −1 is vertical → dy

$$x = y,\ x = \sqrt[3]{y} \rightarrow y = 0, 1$$

$$y = \frac{1}{8} \rightarrow \sqrt[3]{y} = \frac{1}{2} > \frac{1}{8}$$

$$x = -1 \rightarrow \text{right} - \text{left}$$

$$R = \sqrt[3]{y} - (-1) = 1 + \sqrt[3]{y}\ (\text{far})$$

$$r = y - (-1) = 1 + y\ (\text{near})$$

Solution A vertical axis: dy; both radii from x = -1

$R = 1 + \sqrt[3]{y},\ r = 1 + y \Rightarrow \pi\int_0^1 (R^2 - r^2)dy$

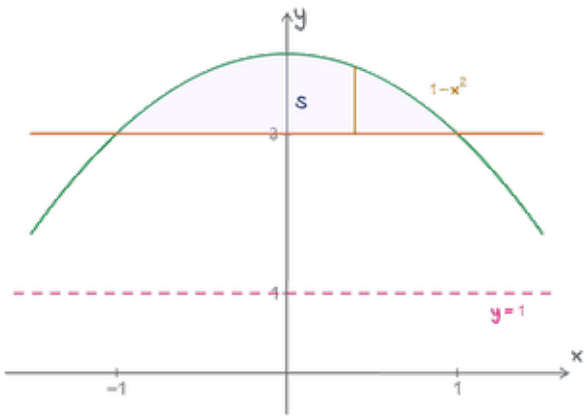
(B) $r^2 - R^2$ (C) $(R - r)^2$ (D) y-axis

(A)

Workbook → vertical axis: dy

PICK THE TOOL: AREA, SLICES, DISC, WASHER

Same region, four questions



Area → $\int_{-1}^{1} ((4 - x^2) - 3)dx$

Square slices → $\int_{-1}^{1} (1 - x^2)^2\,dx$ → no π

Around y = 3 → disc $\pi\int_{-1}^{1} (1 - x^2)^2 dx$

y = 1 → washer $\pi\int_{-1}^{1} ((3 - x^2)^2 - 2^2)dx$

Don't evaluate → ∫

WRAP-UP

R and r start at the axis

Both radii from the axis

R farther, r nearer → can trade places

$V = \pi\int_a^b (R^2 - r^2)dx$; axis x = k → dy

Workbook → comments

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.