8.12 / Volume with Washer Method: Revolving Around Other Axes Watch the lesson ➤

Both radii from the axis: R = farther curve, r = nearer curve

Learning goal

Find the volume of a solid of revolution with washers around another horizontal or vertical line: both radii measured from the axis, far is R, near is r.

Key ideas

01 **Mark the axis** on your sketch first. Both radii are distances from it.

02 R = distance from the axis to the **farther** curve, r = distance to the nearer curve; each is bigger minus smaller coordinate.

03 When the axis is above or to the right of the region, the top curve is the **near** one: "top curve = R" breaks.

04 Horizontal axis $y = k$: dx. Vertical axis $x = k$: curves in y, thickness dy.

05 Read what is asked: an area, a volume with square slices, discs, or washers each need their own integral.

Remember

far is R
The outer radius goes to the curve farther from the axis.

near is r
The inner radius goes to the curve nearer the axis.

shifted axis
An axis $y = k$ or $x = k$ that is not one of the coordinate axes.

Class notes

8.12 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write R and r as distances from the axis: axis $y = 1$, $R = (4 - x^2) - 1$, $r = 3 - 1$.	Do not take R as the top curve without checking where the axis is.
Write, but do not evaluate, means the integral with π, its ends, and $R^2 - r^2$ inside.	Do not measure a radius from the x-axis when the axis is $y = k$.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Measure from the shifted axis

Code V10

✗ Region between $y = 4 - x^2$ and $y = 3$, around $y = 1$: $R = 4 - x^2$, $r = 3$.

✓ $R = (4 - x^2) - 1 = 3 - x^2$, $r = 3 - 1 = 2$: $\pi \int_{-1}^{1} \left((3 - x^2)^2 - 4\right) dx = \frac{32\pi}{5}$.

02 Axis above means the top curve is near

Code V11

✗ Region under $y = \sqrt{x}$, $0 \le x \le 4$, around $y = 3$: $R = 3 - \sqrt{x}$, $r = 3$.

✓ The x-axis is farther from $y = 3$: $R = 3$, $r = 3 - \sqrt{x}$. $V = 24\pi$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

2024 AB6(c) asks for the volume when a region is revolved around a horizontal line that is not the x-axis: R and r measured from that line.

8.12 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Washers around y = k	R = farther curve, r = nearer curve, both measured from $y = k$.	E01 · Practice 1, 2, 3, 5, 6
Washers around x = k	Curves as $x = g(y)$; R and r from $x = k$; integrate in y.	E02 · Practice 4

Example 01

Video · E01

S is the region between $y = 4 - x^2$ and $y = 3$. Find the volume when S is revolved around $y = 1$.

The axis $y = 1$ is below the region. Arrows from $y = 1$ up to each curve.

Example 02

Video · E02

The region between $y = x$ and $y = x^3$ (first quadrant) is revolved around $x = -1$. Which integral gives the volume?

A. $\pi \int_0^1 \left((\sqrt[3]{y} + 1)^2 - (y + 1)^2\right) dy$

B. $\pi \int_0^1 \left((y + 1)^2 - (\sqrt[3]{y} + 1)^2\right) dy$

C. $\pi \int_0^1 (\sqrt[3]{y} - y)^2 \, dy$

D. $\pi \int_0^1 \left(y^{\frac{2}{3}} - y^2\right) dy$

The axis $x = -1$ stands up: curves in y ($x = y$, $x = \sqrt[3]{y}$). Which is farther from $x = -1$?

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.12 / Example answers

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E01 $\frac{32\pi}{5}$

E02 (A)