Lecture Notes · Unit 8 · Lecture 11 AP CED 8.11

Washers

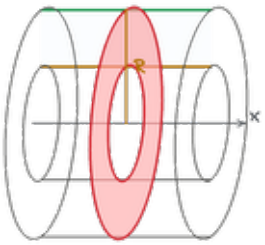
Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: A SOLID WITH A HOLE

Washer Method: x- and y-Axis AP Calculus 8.11

Let's get that 5

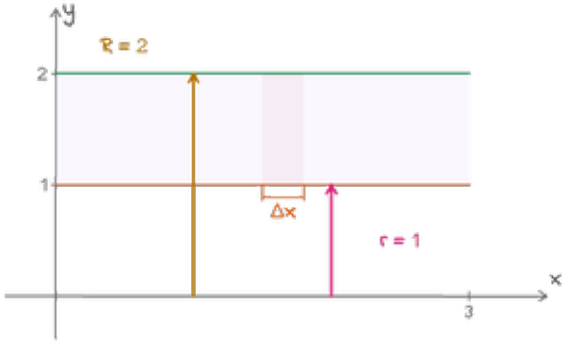
Gap → hole → subtract?



IDEA 1: A WASHER

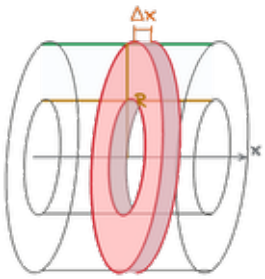
Washers around the x-axis

Band $1 \le y \le 2, 0 \le x \le 3$ → far edge $R = 2$, near edge $r = 1$



1. Every slice is a washer

washer = big disc − hole $= \pi(R^2 - r^2)\,dx$



Touches the axis → $r = 0$ → a disc

$\pi\int_0^3 (2^2 - 1^2)dx = (4-1)3\pi = 9\pi$

$V = \pi\int_a^b (R^2 - r^2)dx$ → square first

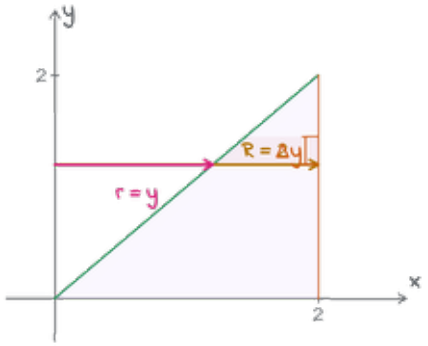
$\pi(R-r)^2 \cdot 3 \rightarrow 3\pi \neq 9\pi$

IDEA 2: AROUND THE Y-AXIS

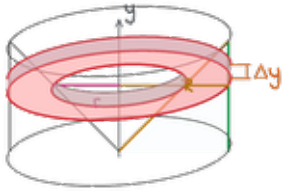
Washers around the y-axis

2. Around the y-axis

$x = y, x = 2, 0 \le y \le 2$ → horizontal strip $R = 2$ $r = y$



washer → $\pi\int_0^2 (4 - y^2)dy = \pi\left(8 - \frac{8}{3}\right) = \frac{16\pi}{3}$



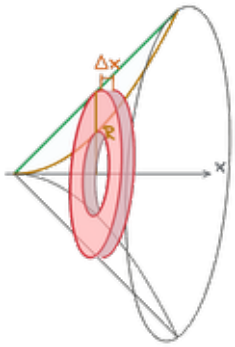
y-axis → $V = \pi\int_c^d (R^2 - r^2)dy$

AP → gap from the axis → washers

PROBLEM 1 OF 2

Line and parabola

S is the region between $y = x$ and $y = x^2$. Find the volume when S is revolved around the x-axis.



Solution Outer and inner radius, squared one at a time

Plan: ends → R, r → $\pi\int (R^2 - r^2)\,dx$

$x = x^2 \to x = 0, 1$

$x = \frac{1}{2} \to \frac{1}{2} > \frac{1}{4} \to R = x,\ r = x^2$

$V = \pi\int_0^1 (x^2 - x^4)dx$

$= \pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}$

Workbook → R and r first

Careful → where points are lost

X Common mistake

$\pi\int_0^1 (x - x^2)^2\,dx = \frac{\pi}{30}$

✓ Do this

$\pi\int_0^1 (x^2 - x^4)\,dx = \frac{2\pi}{15}$

PROBLEM 2 OF 2

Washers around the y-axis

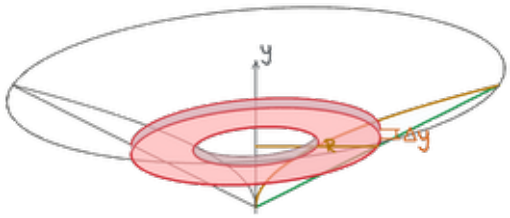
The region between $x = y^2$ and $x = 2y$ is revolved around the y-axis. What is the volume of the solid?

(A) $\frac{64\pi}{15}$

(B) $\frac{16\pi}{15}$

(C) $\frac{8\pi}{3}$

(D) $\frac{32\pi}{5}$



Solution Around the y-axis: R and r in y

Ends, R, r in y → integrate in y

$y^2 = 2y \to 0, 2$

$y = 1 \to 2 > 1 \to R = 2y,\ r = y^2$

$V = \pi\int_0^2 (4y^2 - y^4)dy$

$= \pi\left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64\pi}{15}$

So (A).

Workbook → R, r in y

WRAP-UP

Subtract the hole, an area

Gap from the axis → hole → washers

$\pi\int (R^2 - r^2)\,dx$, not $\pi\int (R - r)^2\,dx$

Touches the axis → r = 0 → disc

R far, r near

y-axis → R, r in y

workbook → comments → 8.12

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.