# 8.11 / Volume with Washer Method: Revolving Around the x- or y-Axis

Watch the lesson ➤

*A washer: pi times (R squared minus r squared), R the farther curve, r the nearer*

## Learning goal

Find the volume of a solid of revolution with a hole using washers around the x- or y-axis: big disc minus small disc, R squared minus r squared.

## Key ideas

01 When the region does not touch the axis, a slice is a **washer**: a disc with a hole.

02 Washer area $= \pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right)$. It is **not** $\pi(R - r)^2$.

03 $R$ is the distance from the axis to the **farther** curve; $r$ is the distance to the nearer curve.

04 If the region touches the axis, $r = 0$ and the washer is a **disc**.

05 Around the $y$-axis: $R$ and $r$ are $x$-values written in $y$, and the thickness is $dy$.

## Remember

**washer**
A disc with a hole: outer radius $R$, inner radius $r$.

**outer radius R**
Distance from the axis to the farther curve.

**inner radius r**
Distance from the axis to the nearer curve.

Class notes

# 8.11 / Write it clearly

*What earns the point on the AP Exam, and the mistakes that lose it.*

## Writing checklist

Notation to use and to avoid

| WRITE | AVOID |
|---|---|
| Write $R$ and $r$ before the integral: $R = x$, $r = x^2$, $V = \pi \int_0^1 (x^2 - x^4)\,dx$. | Do not write $\pi \int (R - r)^2\,dx$. |
| Keep the parentheses: $\pi \int_a^b (R^2 - r^2)\,dx$. | Do not decide $R$ by which curve is on top; decide by which is farther from the axis. |

## Common mistakes

The mistake code is used in the answers and in My Mistakes

### 01 R squared minus r squared

Code V11

✗ Between $y = \sqrt{x}$ and $y = x$, around the $x$-axis: $\pi \int_0^1 (\sqrt{x} - x)^2\,dx = \dfrac{\pi}{30}$.

✓ $\pi \int_0^1 \left((\sqrt{x})^2 - x^2\right)dx = \pi \int_0^1 (x - x^2)\,dx = \dfrac{\pi}{6}$.

### 02 Farther curve is R

Code V11

✗ Between $x = y^2$ and $x = 2y$, around the $y$-axis: $R = y^2$ because it is a square.

✓ At $y = 1$: $2 > 1$, so $x = 2y$ is farther from the $y$-axis. $R = 2y$, $r = y^2$.

**Real AP questions on this topic** Free on AP Central. Try them after the practice.

Washers appear inside released AB volume questions whenever the region does not touch the axis of revolution.

# 8.11 / Guided examples

*Watch, pause, and write the reasoning. Full solutions are in the video.*

## Problem types

Read the problem, name the type, then use its method

| Type | Method | Problems |
|---|---|---|
| Washers around the x-axis | R = farther curve, r = nearer curve: $V = \pi \int_a^b (R^2 - r^2)\,dx$. | E01 · Practice 1, 2, 3, 5, 6 |
| Washers around the y-axis | R and r as $x$ values written in $y$, then $dy$. | E02 · Practice 4 |

### Example 01

Video · E01

$S$ is the region between $y = x$ and $y = x^2$. Find the volume when $S$ is revolved around the $x$-axis.

> The curves meet at $x = 0$ and $x = 1$. Which one is farther from the $x$-axis between them?

### Example 02

Video · E02

The region between $x = y^2$ and $x = 2y$ is revolved around the $y$-axis. Volume?

A. $\frac{64\pi}{15}$

B. $\frac{16\pi}{15}$

C. $\frac{8\pi}{3}$

D. $\frac{32\pi}{5}$

> Around the $y$-axis: $R$ and $r$ are $x$-values in $y$. They meet at $y = 0$ and $y = 2$.

**VIDEO COMPLETE** Check your example answers at the end of this booklet.

# 8.11 / Example answers

*Volume with Washer Method: Revolving Around the x- or y-Axis*

E01 $\frac{2\pi}{15}$

E02 (A)