Lecture Notes · Unit 8 · Lecture 10 AP CED 8.10

Discs, other axes

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO: THE AXIS MOVES

Disc Method · other Axes

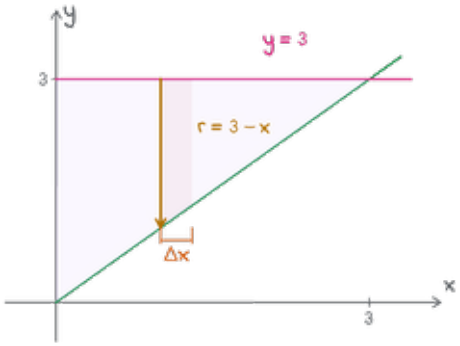
Let's get that 5

x-axis: $r = f(x)$ → axis moves → radius?

IDEA 1: RADIUS = DISTANCE TO THE AXIS

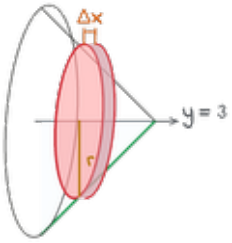
Radius = distance to the axis

$y = x$, $y = 3$ → around $y = 3$ → arrow $3 - x$



1. Radius = distance to the axis

Disc: $r = 3 - x$ → $V = \pi \int_0^3 (3-x)^2 \, dx = 9\pi$



r = distance: axis to curve

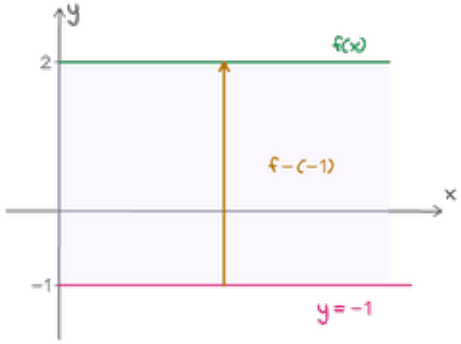
IDEA 2: ABOVE, BELOW, VERTICAL

Axis above, below, or vertical

2. Where is the axis?

Axis $y = k$ above → $r = k - f(x)$

Axis below, $y = -1$ → $r = f(x) - (-1) = f(x) + 1$



$x = k$ → lay the strip down $r = |g(y) - k|$, dy

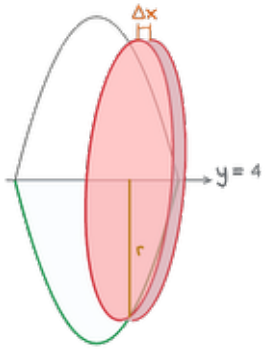
Radius = bigger − smaller ≥ 0

AP → axis like $y = 3$ → measure from that line

PROBLEM 1 OF 2

Around y = 4

R is the region between $y = x^2$ and $y = 4$. Find the volume when R is revolved around $y = 4$.



Solution

Radius = axis - curve; pi times the integral of r squared

$$r = \text{axis} - \text{curve} \to \pi\int r^2\,dx \to \text{full disc}$$

$$x^2 = 4 \to x = \pm 2 \to r = 4 - x^2$$

$$r^2 = 16 - 8x^2 + x^4$$

$$V = 2\pi\int_0^2 \left(16 - 8x^2 + x^4\right)dx$$

$$= 2\pi\left(32 - \frac{64}{3} + \frac{32}{5}\right)$$

$$= 2\pi \cdot \frac{256}{15} = \frac{512\pi}{15}$$

Workbook → arrow from the axis first

Careful → where points are lost

✗ Common mistake

$$\pi\int (x^2)^2\,dx$$

✓ Do this

$r = 4 - x^2$ (axis − curve)

PROBLEM 2 OF 2

Around x = e

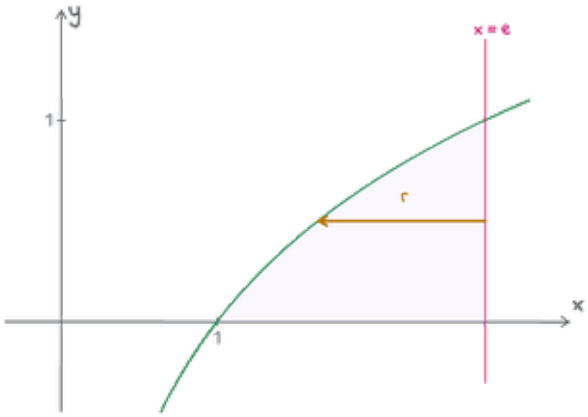
The region bounded by $y = \ln x$, the x-axis, and $x = e$ is revolved around $x = e$. Which integral gives the volume?

(A) $\pi\int_0^1 (e - e^y)^2\,dy$

(B) $\pi\int_1^e (e - x)^2\,dx$

(C) $\pi\int_0^1 e^{2y}\,dy$

(D) $\pi\int_0^1 (e^y - 1)^2\,dy$



Solution

A vertical axis: lay the strip down, radius = axis - curve

Plan: axis x = e is vertical → dy

$$y = \ln x \to x = e^y \to 0 \le y \le 1$$

$e - e^y$ = axis − curve

(B) dx (C) x = 0 (D) x = 1

So (A).

Workbook → horizontal strip, dy

WRAP-UP



Radius = distance to the axis

Radius = distance from the axis

Bigger − smaller → k − f or f − k

Axis x = k → lay the strip down, dy

workbook → comments → 8.11 → hole

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.