8.10 / Volume with Disc Method: Revolving Around Other Axes

Watch the lesson ➤

Radius = the distance from the axis to the curve: bigger minus smaller

Learning goal

Find the volume of a solid of revolution with discs when the axis is another horizontal or vertical line: the radius is the distance from the axis, bigger minus smaller.

Key ideas

01 Draw the **arrow** from the axis to the curve before you write the radius.

02 Radius = distance from the axis = **bigger** coordinate minus smaller coordinate. Axis $y = 4$ above the curve: $r = 4 - f(x)$.

03 The height $f(x)$ itself is the radius only when the axis is the x-axis. When the axis moves, the **radius** changes.

04 Horizontal axis $y = k$: thickness dx. Vertical axis $x = k$: lay the strip down, thickness dy, curves in y.

Remember

axis of revolution
The line $y = k$ or $x = k$ the region spins around.

radius
The distance from the axis to the curve, never negative.

horizontal strip
Used for a vertical axis $x = k$: thickness dy.

Class notes

8.10 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the radius as a distance before the integral: $r = 4 - x^2$ for the axis $y = 4$.	Do not use the height of the curve as the radius when the axis is not the x-axis.
For a vertical axis, show the curve in y and the radius in y: $x = e^y$, $r = e - e^y$.	Do not subtract in the order that gives a negative radius.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The radius starts at the axis

Code V10

✗ Region between $y = 2x^2$ and $y = 2$, around $y = 2$: $r = 2x^2$.

✓ The axis $y = 2$ is above the curve: $r = 2 - 2x^2$, $V = \pi \int_{-1}^{1} (2 - 2x^2)^2 \, dx = \frac{64\pi}{15}$.

02 A vertical axis needs dy

Code V7

✗ Around $x = 4$, under $y = \sqrt{x}$: $\pi \int_0^4 (4 - \sqrt{x})^2 \, dx$.

✓ $x = y^2$, $r = 4 - y^2$, $0 \le y \le 2$: $\pi \int_0^2 (4 - y^2)^2 \, dy = \frac{256\pi}{15}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

Released AB volume questions often move the axis to a line such as y = k; the radius is then a distance from that line.

8.10 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Discs around y = k	Radius = axis − curve (axis above) or curve − axis (axis below), then $\pi \int r^2\,dx$.	E01 · Practice 1, 2, 5, 6
Discs around x = k	Lay the strip down: radius = $\lvert g(y) - k \rvert$, $\pi \int r^2\,dy$.	E02 · Practice 3, 4

Example 01

Video · E01

R is the region between $y = x^2$ and $y = 4$. Find the volume when R is revolved around $y = 4$.

The axis $y = 4$ is above the region. Arrow from the axis down to the curve: $r = 4 - x^2$.

Example 02

Video · E02

The region bounded by $y = \ln x$, the x-axis, and $x = e$ is revolved around $x = e$. Which integral gives the volume?

A. $\pi \int_0^1 (e - e^y)^2\,dy$

B. $\pi \int_1^e (e - x)^2\,dx$

C. $\pi \int_0^1 e^{2y}\,dy$

D. $\pi \int_0^1 (e^y - 1)^2\,dy$

The axis $x = e$ stands up: lay the strip down. $x = e^y$, and the radius runs from the curve to the axis.

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.10 / Example answers

Volume with Disc Method: Revolving Around Other Axes

E01 $\frac{512\pi}{15}$

E02 (A)