Lecture Notes · Unit 8 · Lecture 9 AP CED 8.9

Discs

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: WHERE THE SPHERE FORMULA COMES FROM

Disc Method · x- and y-Axes

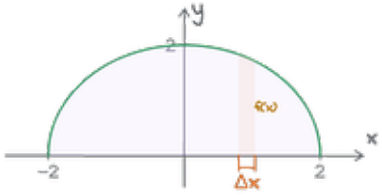
Let's get that 5

Sphere $\frac{4}{3}\pi r^3$ from discs

IDEA 1: A STRIP SPINS INTO A DISC

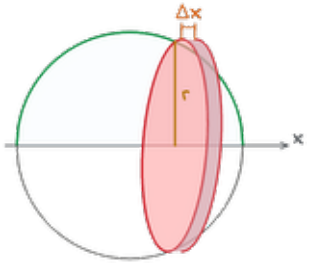
Discs around the x-axis

$\frac{1}{2}$ disk: $y = \sqrt{4 - x^2}$ → strip height $f(x)$



1. A strip spins into a disc

Disc → radius $f(x)$, thickness dx → $\pi f(x)^2\,dx$



Disc → circle → π. 8.7-8.8 square, triangle → no π

on axis → no hole

$V = \pi \int_a^b f(x)^2\,dx$ → disc method

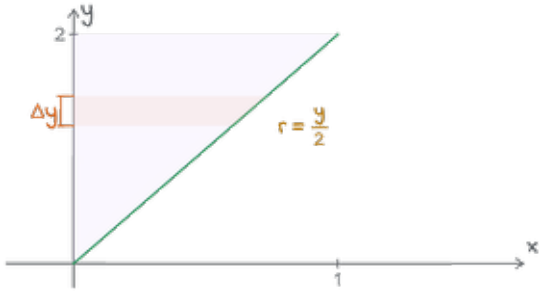
$\pi \int_{-2}^{2} (4 - x^2)\,dx = 16\pi - \frac{16\pi}{3} = \frac{32\pi}{3} = \frac{4}{3}\pi \cdot 2^3$

IDEA 2: AROUND THE Y-AXIS

Discs around the y-axis

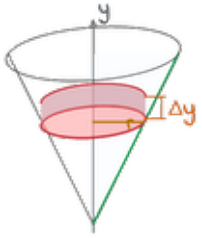
2. Around the y-axis

Lay it down → out from the axis → radius $x = g(y)$, dy



$V = \pi \int_c^d g(y)^2\,dy$

Cone → $\pi \int_0^2 \left(\frac{y}{2}\right)^2 dy = \frac{2\pi}{3}$



AP → region spun around an axis → volume

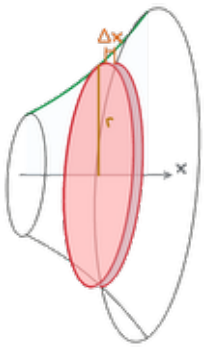
PROBLEM 1 OF 2

Under e to the x over 2

R is the region under $y = e^{\frac{x}{2}}$, above the x-axis, for $0 \le x \le 2$.

(a) Find the area of R.

(b) Find the volume when R is revolved around the x-axis.



Solution

Area: the integral of f; volume: pi times the integral of f squared

Plan: (a) $\int f$ (b) $\pi \int f^2$ → discs

(a)

$$\int_0^2 e^{\frac{x}{2}}\,dx = \left[2e^{\frac{x}{2}}\right]_0^2 = 2e - 2$$

(b)

$$r = e^{\frac{x}{2}} \rightarrow r^2 = e^x$$

$$V = \pi \int_0^2 e^x\,dx = \pi(e^2 - 1)$$

Workbook → area: f, volume: πf^2

Careful → where points are lost

✗ Common mistake

$$\pi \int f\,dx$$

$$\pi\left(\int f\,dx\right)^2$$

✓ Do this

$$\pi \int f(x)^2\,dx$$

PROBLEM 2 OF 2

Around the y-axis

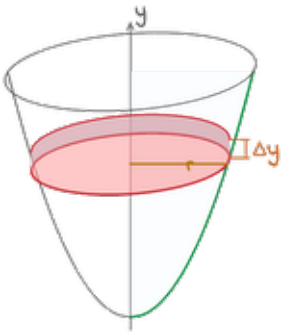
The region bounded by $y = x^2$, $y = 4$, and the y-axis in the first quadrant is revolved around the y-axis. What is the volume of the solid?

(A) 8π

(B) $\frac{32\pi}{5}$

(C) 16π

(D) 4π



Solution

Around the y-axis: radius x = g(y), integrate in y

Plan: radius in y → $\pi \int r^2\,dy$

$$y = x^2 \rightarrow x = \sqrt{y} \rightarrow r^2 = y$$

$$V = \pi \int_0^4 y\,dy = \pi \cdot 8 = 8\pi$$

So (A).

Workbook: y-axis → solve for x first

WRAP-UP

A sphere is a stack of discs

Strip → disc → $\pi r^2\,dx$

x-axis → $V = \pi \int f(x)^2\,dx$

y-axis → lay it down, $V = \pi \int g(y)^2\,dy$

π → only from a circle

workbook → comments → 8.10

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.