8.9 / Volume with Disc Method: Revolving Around the x- or y-Axis

Watch the lesson ➤

A strip spins into a disc: pi r squared dx, or lay the strip down around the y-axis

Learning goal

Find the volume of a solid of revolution around the x- or y-axis with discs: cut perpendicular to the axis, the radius from the axis to the curve, pi r squared times the thickness.

Key ideas

01 Spin a region around an axis. A cut **perpendicular** to the axis is a disc.

02 The radius is the **distance** from the axis to the curve. Disc area $= \pi r^2$; volume $= \int \pi r^2$ times the thickness.

03 Around the x-axis: $r = f(x)$ and $V = \pi \int_a^b f(x)^2\,dx$.

04 Around the y-axis the thickness is dy: solve for x first, so $r = g(y)$, and use y-**ends**.

05 The π comes from the **circle**. Square and triangle slices have no π.

Remember

solid of revolution
The solid made by spinning a region around a line.

disc method
$V = \pi \int r^2$ (thickness), with r from the axis to the curve.

axis of revolution
The line the region spins around.

Class notes

8.9 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the radius first, then the integral: $r = e^{x/2}$, $V = \pi \int_0^2 \left(e^{x/2}\right)^2 dx$.	Do not keep x-ends, or a function of x, when the thickness is dy.
For the y-axis, show the curve solved for x and the y-ends: $x = \sqrt{y}$, $V = \pi \int_0^4 (\sqrt{y})^2 \, dy$.	Do not forget to square the radius.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 y-axis means y

Code V7

✗ Region bounded by $y = x^2$, $y = 4$, the y-axis, around the y-axis: $\pi \int_0^2 (x^2)^2 \, dx$.

✓ $x = \sqrt{y}$, $0 \le y \le 4$: $\pi \int_0^4 y \, dy = 8\pi$.

02 Square the radius

Code V10

✗ Around the x-axis, under $y = x^2$ on $[0, 1]$: $\pi \int_0^1 x^2 \, dx = \frac{\pi}{3}$.

✓ Radius x^2: $\pi \int_0^1 (x^2)^2 \, dx = \frac{\pi}{5}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.
Discs around an axis appear inside released AB volume questions, together with the area of the same region.

8.9 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Discs around the x-axis	Radius $= f(x)$: $V = \pi \int_a^b f(x)^2\,dx$.	E01 · Practice 1, 2, 3, 5
Discs around the y-axis	Write $x = g(y)$, ends in y: $V = \pi \int_c^d g(y)^2\,dy$.	E02 · Practice 4, 6

Example 01

Video · E01

R is the region under $y = e^{\frac{x}{2}}$, above the x-axis, for $0 \le x \le 2$.
(a) Find the area of R.
(b) Find the volume when R is revolved around the x-axis.

> Area takes f. Volume takes πf^2.

Example 02

Video · E02

The region bounded by $y = x^2$, $y = 4$, and the y-axis (first quadrant) is revolved around the y-axis. Volume?

A. 8π
B. $\dfrac{32\pi}{5}$
C. 16π
D. 4π

> Around the y-axis: solve $y = x^2$ for x, and use y from 0 to 4.

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.9 / Example answers

Volume with Disc Method: Revolving Around the x- or y-Axis

E01 (a) $2(e-1)$; (b) $\pi(e^2-1)$

E02 (A)