Lecture Notes · Unit 8 · Lecture 8 AP CED 8.8

Cross sections: semicircles and triangles

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: SAME BASE, NEW SLICES

Cross sections · semicircles and triangles

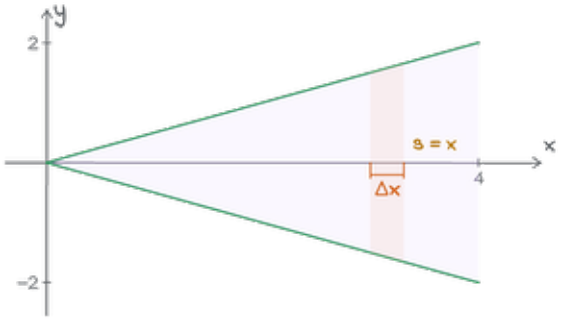
Let's get that 5

Same base → new slices → what changes?

IDEA 1: SEMICIRCLES

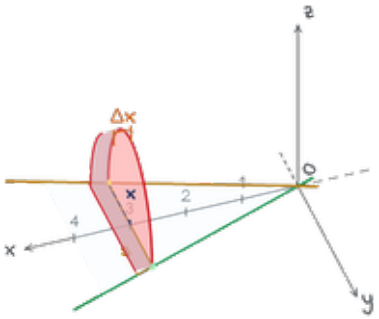
Key ideas

$y = \pm\frac{x}{2}$, $0 \le x \le 4$ → side $s = x$



1. Semicircles

Diameter s → radius $\frac{s}{2}$ → $A = \frac{1}{2}\pi\left(\frac{s}{2}\right)^2 = \frac{\pi}{8}s^2$



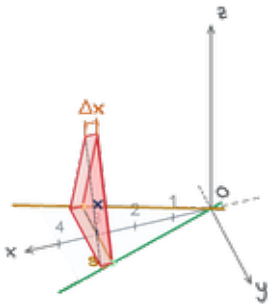
$V = \int_0^4 \frac{\pi}{8}x^2\,dx = \frac{\pi}{8}\cdot\frac{64}{3} = \frac{8\pi}{3}$

IDEA 2: TRIANGLES

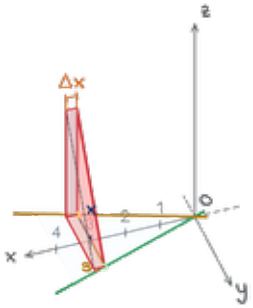
Key ideas

2. Triangles in 3 flavors

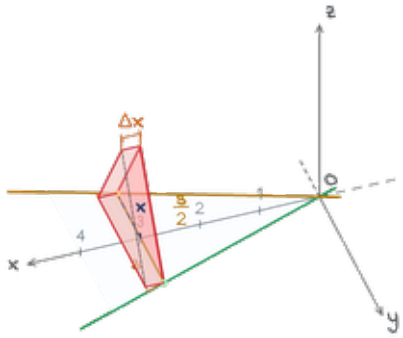
Equilateral, side s → $A = \frac{\sqrt{3}}{4}s^2$



Isosceles right, leg s on base → $A = \frac{1}{2}s^2$



Hypotenuse s on base → height $\frac{s}{2}$ → $A = \frac{1}{4}s^2$



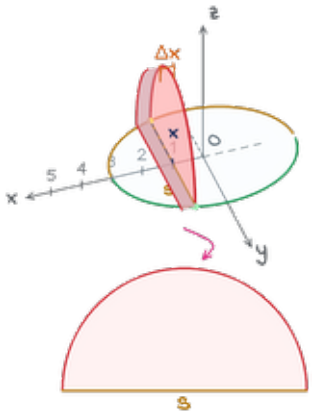
$V = \int_a^b A\,dx$, s = top − bottom

A(x) given → $\int_a^b A(x)dx$ (no extra π)

PROBLEM 1 OF 2

Slices on a disk

The base of a solid is the disk $x^2 + y^2 \le 9$.
(a) Cross sections perpendicular to the x-axis are semicircles. Find the volume.
(b) If they are equilateral triangles, find the volume.



Solution

Side = top - bottom, the shape's area, then the integral

Plan: side → shape's area → $\int_{-3}^{3} A(x)\,dx$

AP: same strip → only area changes

(a)

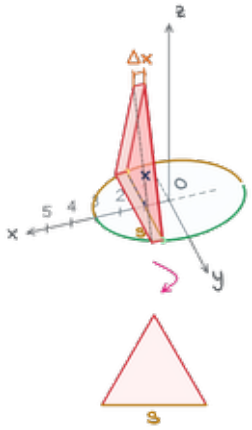
$$s = \sqrt{9-x^2} - \left(-\sqrt{9-x^2}\right) = 2\sqrt{9-x^2}$$

$$A = \frac{\pi}{8}s^2 = \frac{\pi}{8}\cdot 4(9-x^2) = \frac{\pi}{2}(9-x^2)$$

$$V = \frac{\pi}{2}\int_{-3}^{3}(9-x^2)dx = 36\cdot\frac{\pi}{2} = \boxed{18\pi}$$

Solution

Side = top - bottom, the shape's area, then the integral



Plan: same s → triangle

(b)

$$s = 2\sqrt{9-x^2} \rightarrow A = \frac{\sqrt{3}}{4}s^2 = \sqrt{3}(9-x^2)$$

$$V = \sqrt{3}\int_{-3}^{3}(9-x^2)dx = \sqrt{3}\cdot 36 = \boxed{36\sqrt{3}}$$

2 solids, 1 integral. workbook → name the shape

Careful → where points are lost

X Common mistake

Semicircle on strip s

$$A = \frac{1}{2}\pi s^2$$

✓ Do this

$$r = \frac{s}{2} \rightarrow A = \frac{\pi}{8}s^2$$

PROBLEM 2 OF 2

Equilateral triangles

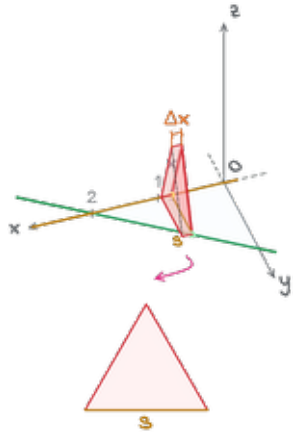
The base is the triangle under $y = 2 - x$ in the first quadrant. Cross sections perpendicular to the x-axis are equilateral triangles. What is the volume of the solid?

(A) $\frac{2\sqrt{3}}{3}$

(B) $\frac{8}{3}$

(C) $\frac{4\sqrt{3}}{3}$

(D) $\frac{\sqrt{3}}{3}$



Solution Side, the equilateral formula, the integral

$$0 \le x \le 2,\ s = 2 - x \to \frac{\sqrt{3}}{4}s^2 \to \int$$

$$A = \frac{\sqrt{3}}{4}(2-x)^2$$

$$V = \frac{\sqrt{3}}{4}\int_0^2 (2-x)^2 dx = \frac{\sqrt{3}}{4} \cdot \frac{8}{3} = \frac{2\sqrt{3}}{3}$$

So (A).

workbook → the formula, every time

WRAP-UP

only the slice's area changes

Same setup: s = top − bottom, $V = \int A\,dx$

Semicircle: $\frac{\pi}{8}s^2$; equilateral $\frac{\sqrt{3}}{4}s^2$

Isosceles right: leg $\frac{1}{2}s^2$, hypotenuse $\frac{1}{4}s^2$

workbook → video → comments → next: 8.9

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.