8.8 / Volumes with Cross Sections: Triangles and Semicircles

Watch the lesson ➤

Same setup, a new area formula: semicircle, equilateral, isosceles right

Learning goal

Find the volume of a solid with semicircle or triangle cross sections: the same stack of slices as squares, with the area formula of the named shape.

Key ideas

01 Same setup as 8.7: ends, the side s (top minus bottom), the **area** of one slice, then $\int A(x)\,dx$.

02 Semicircle: the strip is the **diameter**, so the radius is $\frac{s}{2}$ and $A = \frac{1}{2}\pi\left(\frac{s}{2}\right)^2 = \frac{\pi}{8}s^2$.

03 Equilateral triangle with side s: $A = \frac{\sqrt{3}}{4}s^2$.

04 Isosceles right triangle: with a **leg** in the base, $A = \frac{1}{2}s^2$; with the hypotenuse in the base, $A = \frac{1}{4}s^2$.

05 Name the **shape** before you write its formula. If $A(x)$ is given, integrate it as it is.

Remember

semicircle
Half a circle; here the strip in the base is its diameter.

equilateral triangle
A triangle with three equal sides.

isosceles right triangle
A right triangle with two equal legs.

Class notes

8.8 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Show the radius from the diameter: $s = 2\sqrt{9-x^2}$, $r = \frac{s}{2} = \sqrt{9-x^2}$, $A = \frac{1}{2}\pi r^2$.	Do not use the strip as the radius of a semicircle.
Write the shape's area formula with s before you put in the strip.	Do not use the leg formula when the hypotenuse is in the base.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The strip is the diameter

Code V9

✗ Semicircles on the triangle under $y = 3 - x$: $\int_0^3 \frac{1}{2}\pi(3-x)^2\,dx$.

✓ Radius $\frac{3-x}{2}$: $\int_0^3 \frac{1}{2}\pi\left(\frac{3-x}{2}\right)^2 dx = \frac{9\pi}{8}$.

02 Leg or hypotenuse

Code V9

✗ Hypotenuse s in the base, so $A = \frac{1}{2}s^2$.

✓ With the hypotenuse s in the base, each leg is $\frac{s}{\sqrt{2}}$: $A = \frac{1}{2}\left(\frac{s}{\sqrt{2}}\right)^2 = \frac{1}{4}s^2$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

Released AB volume questions name the cross-section shape in words (squares, rectangles, semicircles, triangles); the strip in the base gives its side.

8.8 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Semicircle cross sections	The side is the diameter: $A = \frac{\pi}{8}s^2$.	E01 · Practice 1, 5
Triangle cross sections	Equilateral $\frac{\sqrt{3}}{4}s^2$; isosceles right: leg $\frac{1}{2}s^2$, hypotenuse $\frac{1}{4}s^2$.	E02 · Practice 2, 3, 4, 6

Example 01

Video · E01

The base of a solid is the disk $x^2 + y^2 \leq 9$.
(a) Cross sections perpendicular to the x-axis are semicircles. Find the volume.
(b) If they are equilateral triangles, find the volume.

At x, the strip goes from $-\sqrt{9 - x^2}$ to $\sqrt{9 - x^2}$. For a semicircle it is the diameter.

Example 02

Video · E02

The base is the triangle under $y = 2 - x$ in the first quadrant. Cross sections perpendicular to the x-axis are equilateral triangles. Volume?

Side $2 - x$; equilateral area $\frac{\sqrt{3}}{4}s^2$.

A. $\frac{2\sqrt{3}}{3}$

B. $\frac{8}{3}$

C. $\frac{4\sqrt{3}}{3}$

D. $\frac{\sqrt{3}}{3}$

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.8 / Example answers

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E01 (a) 18π; (b) $36\sqrt{3}$

E02 (A)