Lecture Notes · Unit 8 · Lecture 7 AP CED 8.7

Cross sections: squares and rectangles

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: A LOAF OF BREAD

Cross sections · squares and rectangles

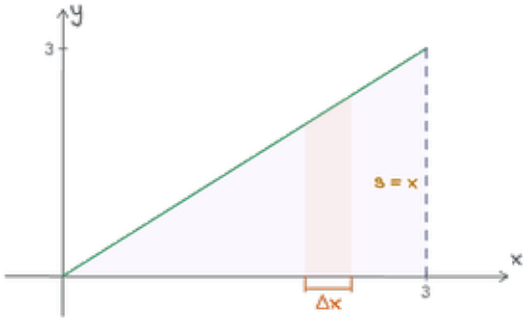
Let's get that 5

Loaf → flat slices → volume?

IDEA 1: SLICE TIMES THICKNESS

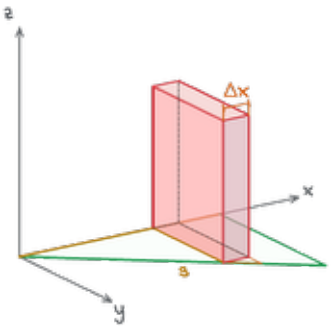
Key ideas

Base: $0 \le y \le x$, $0 \le x \le 3$; squares on it



1. Slice x thickness

Slice at x → side $s = x$ → $A(x) = x^2$ → each x



thickness → $A(x)\,\Delta x$ → $V = \int_0^3 x^2\,dx = 9$

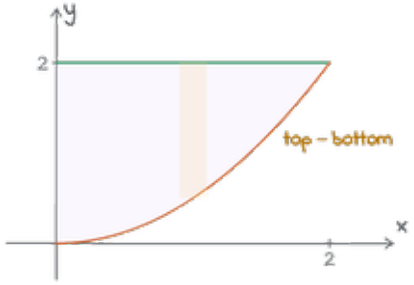
$V = \int_a^b A(x)\,dx$

IDEA 2: SIDE = TOP - BOTTOM

Key ideas

2. The side of a slice

Between two curves → s = top − bottom



Square: $A = s^2 = (\text{top} - \text{bottom})^2$

Rectangle: $A = s \cdot h$, h given → multiply no π

s = top − bottom first, then square it

Horizontal → right − left dy

AP → a base, and cross sections standing on it

PROBLEM 1 OF 2

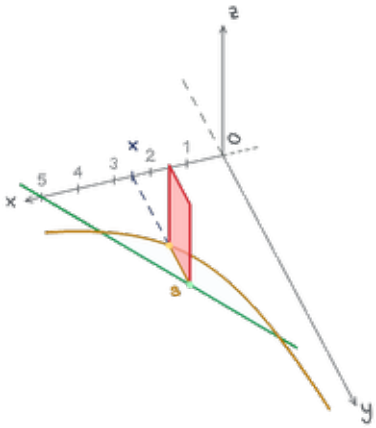
Squares on a curved base

The base of a solid is the region R between $y = 5 - x$ and $y = \frac{4}{x}$.

(a) Find the area of R.

(b) Cross sections perpendicular to the x-axis are squares. Find the volume.

(c) If they are rectangles of height $3x$, write, but do not evaluate, an integral expression for the volume.



Solution

Side = top - bottom, area = side squared, then the integral

Plan: ends → side → area → $\int$

(a)

$$5-x=\frac{4}{x} \rightarrow x^2-5x+4=0 \rightarrow x=1,\ 4$$

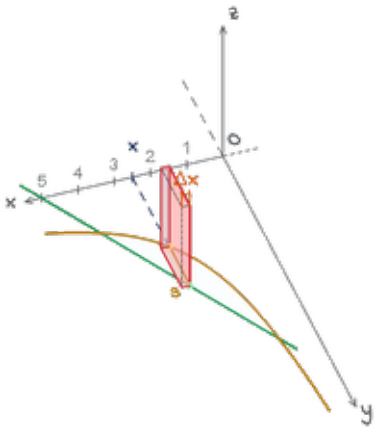
$$\int_1^4\left(5-x-\frac{4}{x}\right)dx=\left[5x-\frac{x^2}{2}-4\ln x\right]_1^4$$

$$=(12-4\ln 4)-\frac{9}{2}=\frac{15}{2}-8\ln 2$$

Area shaky? → workbook, 8.4

Solution

Side = top - bottom, area = side squared, then the integral



Plan: side → square it → $\int$

(b)

$$s=5-x-\frac{4}{x} \rightarrow A=\left(5-x-\frac{4}{x}\right)^2$$

$$A=x^2-10x+33-\frac{40}{x}+\frac{16}{x^2}$$

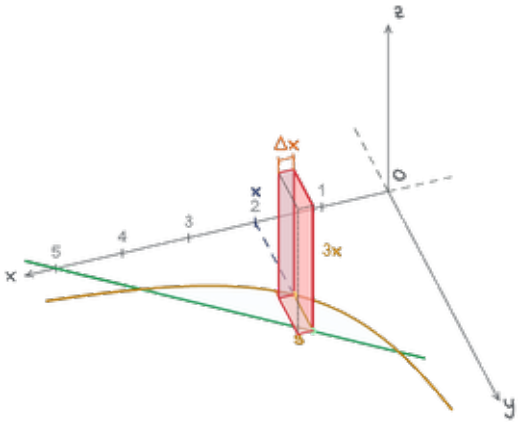
$$V=\int_1^4 A\,dx=\left[\frac{x^3}{3}-5x^2+33x-40\ln x-\frac{16}{x}\right]_1^4$$

$$=\left(\frac{208}{3}-40\ln 4\right)-\frac{37}{3}=57-80\ln 2$$

Area of R ≠ slice area

Solution

Side = top - bottom, area = side squared, then the integral



Plan: (c) $A=s\cdot 3x$ → $\int$

(c)

$$V=\int_1^4 3x\left(5-x-\frac{4}{x}\right)dx$$

Workbook → all four steps on paper

Careful → where points are lost

X Common mistake

$$5-x-\frac{4}{x}$$

$$(5-x)^2-\left(\frac{4}{x}\right)^2$$

✓ Do this

$(5-x-\frac{4}{x})^2$

PROBLEM 2 OF 2

Rectangles of height 3

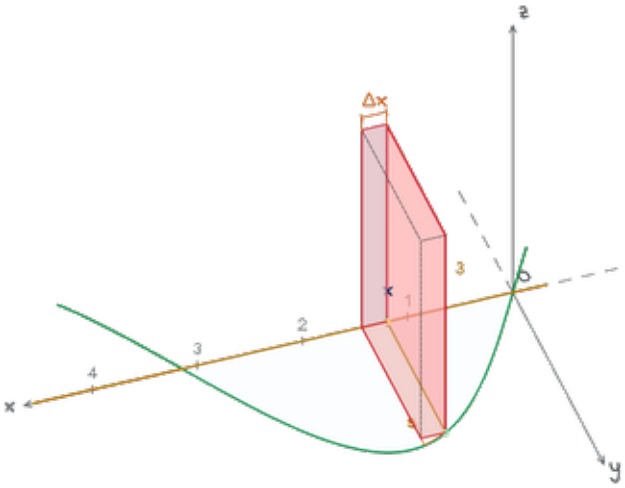
The base is the region between $y=\sin x$ and the x-axis, $0 \le x \le \pi$. Cross sections perpendicular to the x-axis are rectangles of height 3. What is the volume of the solid?

(A) 6

(B) $\frac{\pi}{2}$

(C) 2

(D) $\frac{9\pi}{2}$



Solution

Rectangle area = side times height; then the integral

Plan: $A = s \cdot 3 \to \int$

$s = \sin x$, $h = 3 \to A(x) = 3\sin x$

$V = \int_0^{\pi} 3\sin x\,dx \to$ 1 arch $\to 3 \cdot 2 = 6$

So (A).

Workbook: rectangles → side × height

WRAP-UP

Flat slices build a volume

Volume $= \int_a^b A(x)\,dx$

Side = top − bottom (or right − left)

Square → s^2; rectangle → $s \cdot h$

Workbook → video → comments → next: 8.8

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.