8.7 / Volumes with Cross Sections: Squares and Rectangles

Watch the lesson ➤

One slice times its thickness: the side is top minus bottom, square it or times the height

Learning goal

Find the volume of a solid with square or rectangular cross sections: the strip in the base is one side of the slice, its area times the thickness is stacked by an integral.

Key ideas

01 Volume $= \int_a^b A(x)\,dx$: the **area of one slice** times its thickness dx, stacked from a to b.

02 The strip in the base, top minus bottom, is one **side** s of the slice. Four steps: ends, side, area, integral.

03 A square slice has area s^2. A rectangle slice has area s times the **height** the problem gives.

04 Slices perpendicular to the x-axis use dx; slices perpendicular to the y-axis use dy and a side in y.

05 No slice here is a circle, so there is **no** π. The area of the base is not the volume.

Remember

cross section
The flat slice you see when the solid is cut perpendicular to an axis.

base
The flat region in the xy-plane that the solid stands on.

perpendicular to the x-axis
Slices that stand up across the x-axis: each one sits at one x, thickness dx.

Class notes

8.7 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Name the side, then the slice area: $s = 5 - x - \frac{4}{x}$, $A(x) = s^2$, then $\int_1^4 A(x)\,dx$.	Do not put π in front of a square or rectangle slice.
Write, but do not evaluate, means the integral with its ends and the area inside, and nothing more.	Do not give the area of the base as the volume.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 No circle, no pi

Code V9

✗ Squares on the region under $y = 2x$, $0 \le x \le 1$: $\pi \int_0^1 (2x)^2\,dx$.

✓ $\int_0^1 (2x)^2\,dx = \frac{4}{3}$. The slices are squares.

02 Square the side

Code V9

✗ Squares on the region under $y = \frac{1}{x}$, $1 \le x \le 3$: $\int_1^3 \frac{1}{x}\,dx = \ln 3$.

✓ Side $s = \frac{1}{x}$, area s^2: $\int_1^3 \frac{1}{x^2}\,dx = \frac{2}{3}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

2024 AB6(b) asks for the volume of a solid with rectangular cross sections on a region between two curves.

8.7 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Square cross sections	Side s = top − bottom, then $\int s^2\,dx$.	E01 · Practice 1, 2, 4, 6
Rectangle cross sections	Side on the base times the height the problem gives, then integrate.	E02 · Practice 3, 5

Example 01

Video · E01

The base R is the region between $y = 5 - x$ and $y = \frac{4}{x}$.
(a) Find the area of R.
(b) Cross sections perpendicular to the x-axis are squares. Find the volume.
(c) If they are rectangles of height $3x$, write, but do not evaluate, the volume.

The curves meet at $x = 1$ and $x = 4$. The side of each slice is the strip $5 - x - \frac{4}{x}$.

Example 02

Video · E02

The base is the region between $y = \sin x$ and the x-axis, $0 \le x \le \pi$. Cross sections perpendicular to the x-axis are rectangles of height 3. Volume?

A. 6
B. $\frac{\pi}{2}$
C. 2
D. $\frac{9\pi}{2}$

Side $\sin x$, height 3: the slice area is $3 \sin x$.

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.7 / Example answers

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E01 (a) $\frac{15}{2} - 8\ln 2$; (b) $57 - 80\ln 2$; (c) $\int_1^4 3x\left(5 - x - \frac{4}{x}\right)dx$

E02 (A)