Lecture Notes · Unit 8 · Lecture 6 AP CED 8.6

Area with many crossings

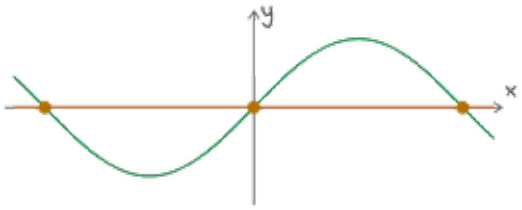
Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: WHEN TOP AND BOTTOM SWAP

Area · 2 curves, 3 crossings

Let's get that 5

2 curves · 3 crossings



IDEA 1: SPLIT AT EACH CROSSING

Key ideas

$y = x^3, y = x \rightarrow$ 3 crossings

1. Split at every crossing

$[-1,0] \rightarrow x^3$ on top $\rightarrow \int_{-1}^{0} (x^3 - x)\,dx = \frac{1}{4}$

$[0,1] \rightarrow x$ on top $\rightarrow \int_{0}^{1} (x - x^3)\,dx = \frac{1}{4}$

Area $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

IDEA 2: WHY ONE INTEGRAL FAILS

Key ideas

2. one integral cancels

$\int_{-1}^{1} (x^3 - x)\,dx = \frac{1}{4} - \frac{1}{4} = 0$

Area $= \int_{a}^{b} |f(x) - g(x)|\,dx$

→ every crossing → top − bottom on each

dy strips → right − left, same split

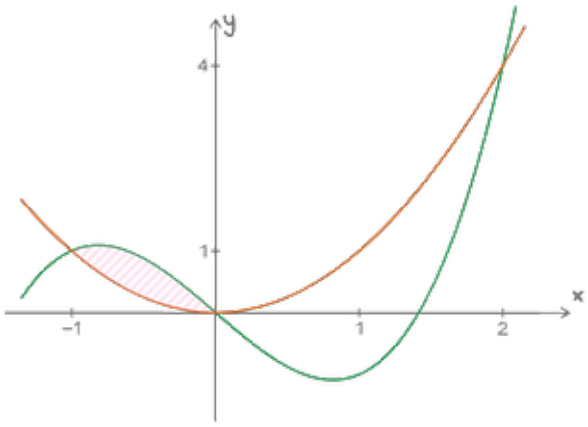
$\int_{a}^{b} |f - g|\,dx \rightarrow$ store the crossings

AP: 3 crossings → one $\int_{a}^{b} (f - g)\,dx$ fails

PROBLEM 1 OF 2

Cubic and parabola

Find the total area of the regions between $y = x^3 - 2x$ and $y = x^2$.



Solution

Every crossing, then top - bottom on each piece

Plan: crossings → tops → add

$x^3 - 2x = x^2 \rightarrow x(x-2)(x+1) = 0$

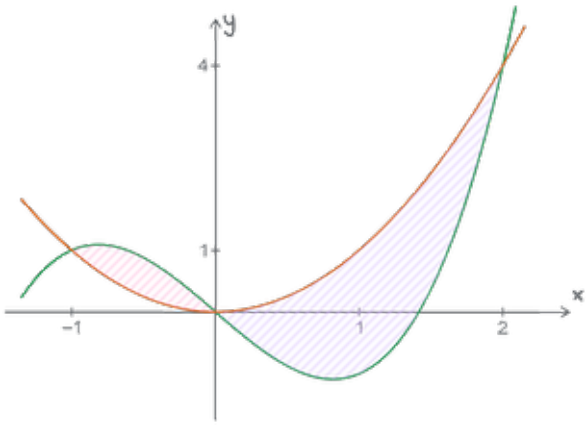
Crossings: $x = -1, 0, 2 \rightarrow$ two pieces

$x = -\frac{1}{2} \rightarrow \frac{7}{8} > \frac{1}{4} \rightarrow \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx = \frac{5}{12}$

Next piece: yours

Solution

Every crossing, then top - bottom on each piece



Plan: second piece → add

$x = 1 \rightarrow 1 > -1 \rightarrow$ top is x^2

$$\int_0^2 \left(x^2 - x^3 + 2x\right) dx = \frac{8}{3} - 4 + 4 = \frac{8}{3}$$

$$\frac{5}{12} + \frac{32}{12} = \frac{37}{12}$$

Workbook: crossings, tops, add

Careful

X Common mistake

$y = x^3 - 2x, y = x^2$

$$\int_{-1}^2 \left(x^3 - x^2 - 2x\right) dx = -\frac{9}{4}$$

$$\left|-\frac{9}{4}\right| = \frac{9}{4} \rightarrow \text{canceled}$$

✓ Do this

Split at 0: $\frac{5}{12} + \frac{8}{3} = \frac{37}{12}$

PROBLEM 2 OF 2

Sine and cosine

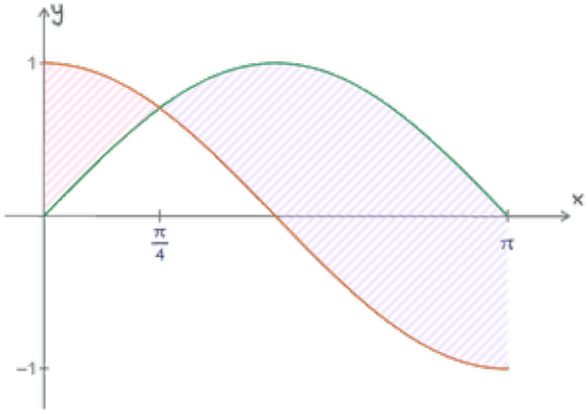
What is the area of the region between $y = \sin x$ and $y = \cos x$ for $0 \le x \le \pi$?

(A) 0

(B) 2

(C) $2\sqrt{2}$

(D) $\sqrt{2}$



Solution Split where sin = cos

Split → add

$$\sin x = \cos x \rightarrow x = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx = \sqrt{2} - 1$$

$$\int_{\frac{\pi}{4}}^{\pi} (\sin x - \cos x) dx = 1 + \sqrt{2}$$

$$\left(\sqrt{2} - 1\right) + \left(1 + \sqrt{2}\right) = 2\sqrt{2}$$

So (C).

Workbook: crossings first

WRAP-UP



One integral of top − bottom? No

Find every crossing

Top − bottom on each piece, then add

One $\int_{-1}^{1}(f-g)\,dx$ → the pieces cancel

Workbook → video → comments → next: 8.7

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.