8.6 / Finding the Area Between Curves That Intersect at More Than Two Points

Watch the lesson ➤

Split at every crossing, top minus bottom on each piece, then add

Learning goal

Find the total area between curves that cross more than once: every crossing, top minus bottom on each piece, then add, so that no piece cancels.

Key ideas

01 Find **every** crossing first: solve $f(x) = g(x)$ and keep the solutions in the interval.

02 On each piece the top curve can change. Use top minus bottom on each piece, then **add** the pieces.

03 One integral of $f - g$ across a crossing lets the pieces **cancel**: it gives a net value, not the area.

04 The total area is $\int_a^b |f(x) - g(x)|\,dx$. With a calculator, integrate the **absolute value** at once, and store the crossings without rounding.

Remember

crossing
An x where the curves meet and the top curve changes.

total area
The sum of the areas of all the pieces, every piece positive.

net integral
$\int_a^b (f - g)\,dx$: pieces where g is on top count negative.

Class notes

8.6 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
List the crossings, then write one integral per piece with its own top: $\int_{-1}^{0}(f-g)\,dx + \int_{0}^{2}(g-f)\,dx$.	Do not integrate $f-g$ across a crossing and call it the area.
With a calculator, write $\int_a^b \lvert f(x)-g(x)\rvert\,dx$ and give three decimal places.	Do not round a crossing found on a calculator before you use it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 One integral lets pieces cancel

Code V8

✗ Area between $y = x^3$ and $y = 9x$: $\int_{-3}^{3}(x^3 - 9x)\,dx = 0$.

✓ Crossings -3, 0, 3: $2\int_{0}^{3}(9x - x^3)\,dx = \frac{81}{2}$.

02 Every crossing

Code V8

✗ $\sin x = \cos x$ only matters at the ends, so the area on $[0, \pi]$ is $\left\lvert\int_{0}^{\pi}(\sin x - \cos x)\,dx\right\rvert = 2$.

✓ They cross at $\frac{\pi}{4}$: $(\sqrt{2}-1) + (1+\sqrt{2}) = 2\sqrt{2}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

Released AB questions with curves that cross several times usually allow a calculator: integrate the absolute value of the difference, with the crossings stored.

8.6 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Area with several crossings	Every crossing ? top on each piece ? add the pieces.	E01, E02 · Practice 1, 3, 5, 6
Net integral vs. area	One integral adds signed pieces; the area adds their sizes.	Practice 2, 4

Example 01

Video · E01

Find the total area of the regions between $y = x^3 - 2x$ and $y = x^2$.

> $x^3 - x^2 - 2x = x(x+1)(x-2)$: three crossings. Test one x in each piece.

Example 02

Video · E02

What is the area between $y = \sin x$ and $y = \cos x$ for $0 \le x \le \pi$?

A. 0
B. 2
C. $2\sqrt{2}$
D. $\sqrt{2}$

> Where does $\sin x = \cos x$ on $[0, \pi]$? Split there.

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.6 / Example answers

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E01 $\frac{37}{12}$

E02 (C)