Lecture Notes · Unit 8 · Lecture 5 AP CED 8.5

Area between curves (dy)

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: WHEN VERTICAL STRIPS GET AWKWARD

Area with dy · 1 dy integral or 2 dx integrals?

8.4: vertical → top − bottom dx

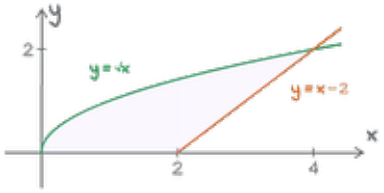
Bottom edge changes: 1 integral → 2?

Today: lay it down, x in y, ends in y

IDEA 1: LAY THE STRIP DOWN

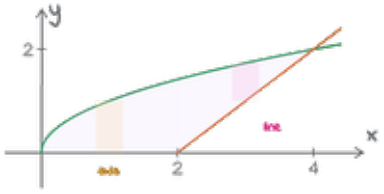
Lay the strip down

Under $y = \sqrt{x}$, above $y = x - 2$ and the x-axis



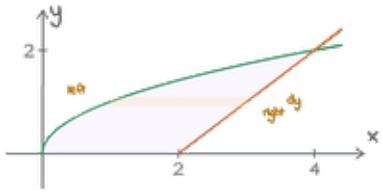
1. Lay the strip down

Vertical $x < 2$ bottom axis, $x > 2$ bottom line



$$\int_0^2 \sqrt{x}\,dx + \int_2^4 (\sqrt{x} - (x-2))\,dx$$

Horizontal → right − left dy



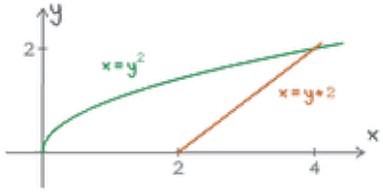
One strip shape → one integral

IDEA 2: EVERYTHING IN Y

x as a function of y

2. x as a function of y

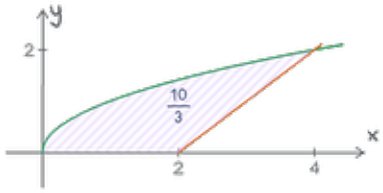
$y = \sqrt{x} \rightarrow x = y^2$; $y = x - 2 \rightarrow x = y + 2$



Only $y \geq 0 \rightarrow \sqrt{x}$ is the top half

Ends in $y \rightarrow 0$ to 2 $\left(y^2 = y + 2\right)$

$$\int_0^2 \left((y+2) - y^2\right) dy = 2 + 4 - \frac{8}{3} = \frac{10}{3}$$



Area = $\int_c^d (\text{right} - \text{left})\,dy$

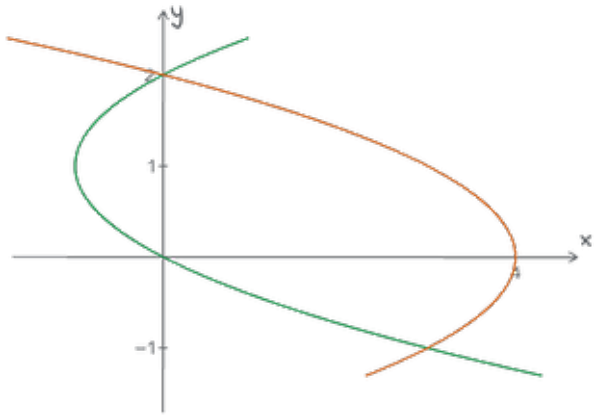
PROBLEM 1 OF 2

Two sideways parabolas

R is the region between $x = y^2 - 2y$ and $x = 4 - y^2$.

(a) Find the area of R.

(b) Find the area of the part of R below $y = 1$.



Solution

Ends in y where they meet; right - left (test one y)

Plan: ends in y → right (test one y) → $\int dy$

(a)

$$y^2 - 2y = 4 - y^2 \rightarrow y^2 - y - 2 = 0$$

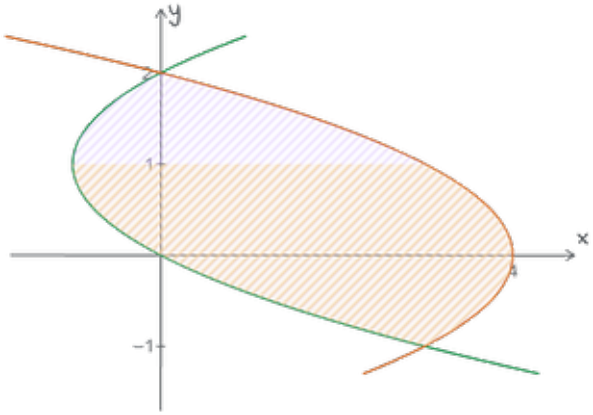
$$(y-2)(y+1) = 0 \rightarrow y = -1, 2$$

$y = 0 \rightarrow 4 > 0 \rightarrow$ right is $4 - y^2$

$$(4 - y^2) - (y^2 - 2y) = 4 + 2y - 2y^2$$

$$A = \int_{-1}^{2} (4 + 2y - 2y^2)\,dy$$

Solution Finish (a); same strips below y = 1



Plan: finish (a), then $y < 1$

$$\int_{-1}^{2} (4 + 2y - 2y^2)\,dy = \left[4y + y^2 - \frac{2}{3}y^3\right]_{-1}^{2}$$

$$= \frac{20}{3} - \left(-\frac{7}{3}\right) = 9$$

(b)

$$\int_{-1}^{1} (4 + 2y - 2y^2)\,dy$$

$$= \left(4 + 1 - \frac{2}{3}\right) - \left(-4 + 1 + \frac{2}{3}\right) = \frac{20}{3}$$

Workbook: ends in y first

Careful → where points are lost

X Common mistake

L − R = −9

x = −1, 4 with dy

✓ Do this

right − left → 9

y = −1, 2

PROBLEM 2 OF 2

Which integral

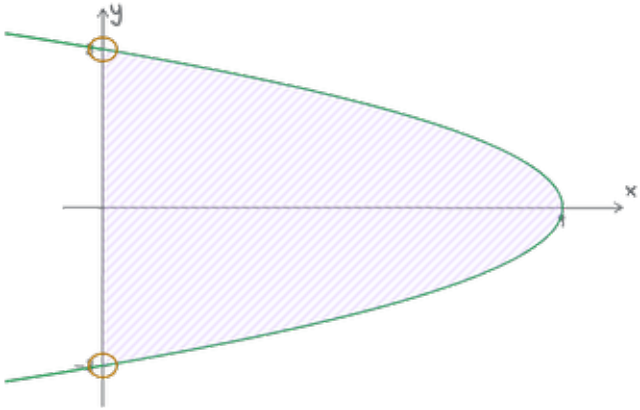
Which integral gives the area of the region between $x = 9 - y^2$ and the y-axis?

(A) $\int_{-3}^{3}(9-y^2)\,dy$

(B) $\int_{0}^{9}(9-y^2)\,dy$

(C) $\int_{-3}^{3}(y^2-9)\,dy$

(D) $\int_{0}^{3}(9-y^2)\,dy$



Solution

Right - left in y, ends in y; each wrong choice in one line

Plan: right − left, ends in y

Right $9 - y^2$, left $x = 0 \to y = \pm 3$

(B) x ends (C) left − right (D) top half

So (A) → $\int_{-3}^{3}\left(9-y^2\right)dy$

Workbook: y ends, not x ends

WRAP-UP

One integral? Yes

Same left and right edges → 1 integral, not 2

Area = $\int_{c}^{d}(\text{right} - \text{left})\,dy$

Ends → y values

Workbook · Next → 8.6: curves that cross

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.