8.5 / Finding the Area Between Curves Expressed as Functions of y Watch the lesson ➤

Lying strips: the integral of right minus left, dy, ends in y

Learning goal

Find the area between curves with horizontal strips: write each curve as x in terms of y, use y-ends, and integrate right minus left; choose dx or dy.

Key ideas

01 A strip can lie down at height y: its length is **right** minus left, its thickness is dy. Area $= \int_c^d (\text{right} - \text{left})\, dy$.

02 With dy, everything is in y: each curve as $x =$ (a function of y), and the ends are **y-values**.

03 Switching x and y is not just renaming letters: solve for x, and with a square root pick the **branch** on the correct side.

04 Choose the strip: if the left and right edges are single curves, lie the strip down (dy); if the top and bottom are, stand it up (dx). One integral with dy can replace a **split** with dx.

Remember

horizontal strip
A strip lying down: length right − left, thickness dy.

in terms of y
A curve written as $x = g(y)$.

y-ends
The lowest and highest y of the region.

Class notes

8.5 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the curves as $x = \ldots$ first, then the integral with y-ends and dy: $\int_{-1}^{2} \big((4 - y^2) - (y^2 - 2y)\big)\, dy$.	Do not keep x-ends with dy.
Test one y between the ends to see which curve is on the right.	Do not mix x and y in one integrand.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 dy means y-ends

Code V7

✗ Region between $x = 9 - y^2$ and the y-axis: $\int_0^9 (9 - y^2)\, dy$.

✓ $9 - y^2 = 0$ at $y = \pm 3$: $\int_{-3}^{3} (9 - y^2)\, dy = 36$.

02 Right minus left

Code V6

✗ Between $x = y^2$ and $x = y$: $\int_0^1 (y^2 - y)\, dy$.

✓ At $y = \frac{1}{2}$: $\frac{1}{2} > \frac{1}{4}$, so $x = y$ is on the right. $\int_0^1 (y - y^2)\, dy = \frac{1}{6}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released AB free-response part asks only for horizontal strips; they appear as one choice inside area and volume questions.

8.5 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Area with horizontal strips	x as a function of y, ends in y, then $\int_c^d (\text{right} - \text{left})\,dy$.	E01 · Practice 1, 2, 3, 6
Choose the strip	Sides are curves ? lying strips; top and bottom are curves ? standing strips.	E02 · Practice 4, 5

Example 01

Video · E01

R is the region between $x = y^2 - 2y$ and $x = 4 - y^2$.
(a) Find the area of R.
(b) Find the area of the part of R below $y = 1$.

> Find the y-values where the curves meet. Test one y for the right curve.

Example 02

Video · E02

Which integral gives the area of the region between $x = 9 - y^2$ and the y-axis?

A. $\int_{-3}^{3}(9 - y^2)\,dy$
B. $\int_{0}^{9}(9 - y^2)\,dy$
C. $\int_{-3}^{3}(y^2 - 9)\,dy$
D. $\int_{0}^{3}(9 - y^2)\,dy$

> The region touches the y-axis where $9 - y^2 = 0$. Those are the ends.

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.5 / Example answers

Finding the Area Between Curves Expressed as Functions of y

E01 (a) 9; (b) $\frac{20}{3}$

E02 (A)