Lecture Notes · Unit 8 · Lecture 4 AP CED 8.4

Area between curves (dx)

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO: THE LAND BETWEEN TWO CURVES

Area Between Curves AP Calculus 8.4

Land: top $y = 4 - x$, bottom $y = x$, left $x = 0$

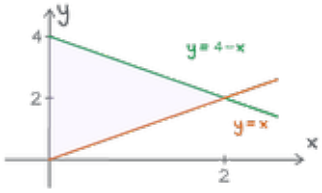
Unit 6: 1 curve to the axis. Two curves?

Today: one strip, top − bottom, the ends

IDEA 1: TOP MINUS BOTTOM

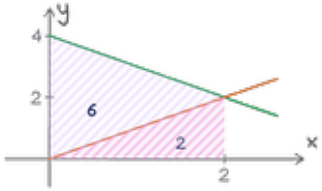
Top − bottom: one strip

Top $y = 4 - x$, bottom $y = x$



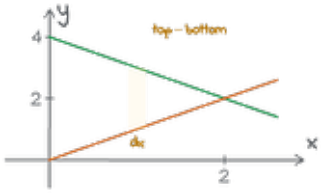
1. Top − bottom

Under top 6 − under bottom 2 = 4



$$\int_0^2 (4-x)\,dx - \int_0^2 x\,dx = \int_0^2 ((4-x)-x)\,dx$$

$(\text{top} - \text{bottom})\,dx$ → thin rectangle = strip



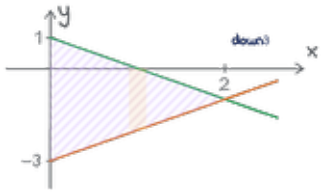
$\int$ all strips $0 \le x \le 2$

IDEA 2: BELOW THE AXIS, AND THE ENDS

Below the axis · the ends

2. Below the axis too

Both down 3 → $y = 1 - x$, $y = x - 3$

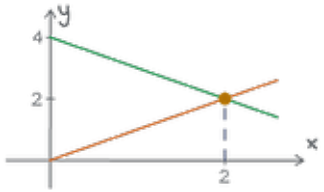


$(1 - x) - (x - 3) = 4 - 2x$ → same area 4

A strip never looks at the axis

3. The ends

Meet: $4 - x = x \rightarrow x = 2$



$$A = \int_a^b (\text{top} - \text{bottom})\,dx$$

a, b → intersection or given: $x = 0$

Edge switches curves → split there

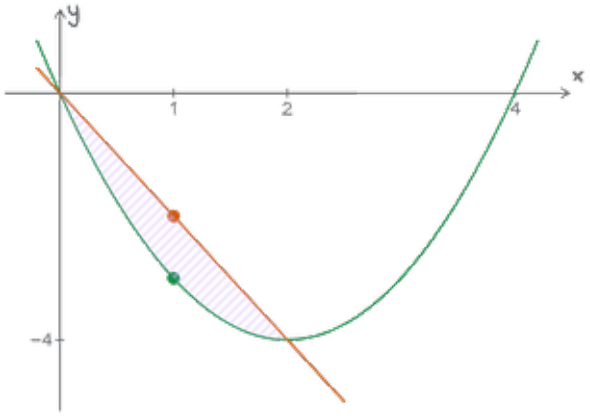
PROBLEM 1 OF 2

A region below the axis

R is the region between $y = x^2 - 4x$ and $y = -2x$.

(a) Find the area of R.

(b) Write an integral for the area of the part of R left of $x = 1$. Then find its value.



Solution

Ends where they meet; top - bottom (test one x)

Plan: ends → top (test one x) → $\int$

(a)

$x^2 - 4x = -2x \rightarrow x(x-2) = 0 \rightarrow x = 0, 2$

$x = 1 \rightarrow -2 > -3 \rightarrow$ top is $-2x$

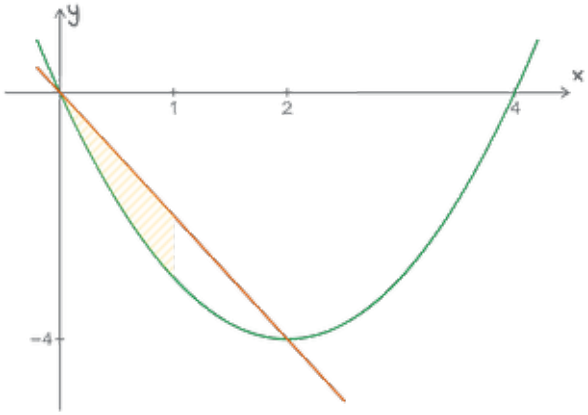
$$\int_0^2 \left(-2x - \left(x^2 - 4x\right)\right)dx = \int_0^2 \left(2x - x^2\right)dx$$

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$

R below axis → area > 0

Solution

Same strips, 0 to 1



(b)

Plan: same strips, 0 to 1

$$\int_0^1 \left(2x - x^2\right)dx$$

2024 AB Q6(a) → "do not evaluate" → stop here

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

Half of R → symmetric about $x = 1$

Workbook: test one x first

Careful → where points are lost

X Common mistake

$\left(x^2 - 4x\right) - (-2x) \rightarrow -\frac{4}{3}$

✓ Do this

$x = 1 \rightarrow -2x$ on top → $\frac{4}{3}$

PROBLEM 2 OF 2

Root and line

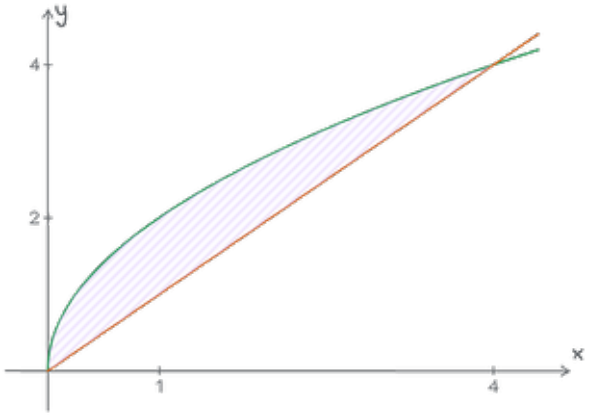
What is the area of the region between $y = 2\sqrt{x}$ and $y = x$?

(A) $\frac{8}{3}$

(B) $-\frac{8}{3}$

(C) $\frac{4}{3}$

(D) $\frac{16}{3}$



Solution Ends, top, integral

Plan: ends → top → $\int$

$2\sqrt{x} = x \rightarrow 4x = x^2 \rightarrow x = 0,\ 4$

$x = 1,\ 2 > 1 \rightarrow$ top $2\sqrt{x}$

$$\int_0^4 (2\sqrt{x} - x)\,dx = \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^4$$

$$= \frac{32}{3} - 8 = \frac{8}{3}$$

So (A).

Workbook: ends, top, integral

WRAP-UP

How big? Four

$\text{Area} = \int_a^b (\text{top} - \text{bottom})\,dx$

Below the axis → still top − bottom

Ends → where they meet; top → test one x

Workbook · Next → 8.5: sideways strips

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.