8.4 / Finding the Area Between Curves Expressed as Functions of x Watch the lesson ➤

Vertical strips: the integral of top minus bottom, the ends where the curves meet

Learning goal

Find the area between two curves with vertical strips: the ends where the curves meet, top minus bottom, even below the x-axis.

Key ideas

01 A **strip** stands up at x: its length is top minus bottom, its thickness is dx. The area adds the strips: $\int_a^b (\text{top} - \text{bottom})\, dx$.

02 The ends a and b are given, or they are where the curves **meet**: solve top = bottom.

03 Test **one** x between the ends to see which curve is on top.

04 Top minus bottom is never negative, even when both curves are **below** the x-axis. No absolute values are needed.

05 If the top or bottom curve changes in the middle, split the integral at that x. "Write, but do not evaluate" asks only for the **integral**.

Remember

strip
A thin rectangle of length top − bottom and thickness dx.

region
The part of the plane between the curves, from one end to the other.

bounded by
The curves and lines that make the edges of the region.

Class notes

8.4 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the integral with its ends, the order top minus bottom, and dx: $\int_0^2 \big((-2x)-(x^2-4x)\big)\,dx$.	Do not put absolute values around a curve because it is below the axis; top minus bottom already works.
Show where the curves meet before you use those x-values as ends.	Do not evaluate an integral the problem says not to evaluate.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Top minus bottom, below the axis too

Code V6

✗ Both curves are below the axis, so the area is $\int_0^2 \big((x^2-4x)-(-2x)\big)\,dx = -\frac{4}{3}$.

✓ At $x=1$: $-2>-3$, so $-2x$ is on top. $\int_0^2 (2x-x^2)\,dx = \frac{4}{3}$.

02 Ends where the curves meet

Code V7

✗ Region between $y=x^2$ and $y=4x$: $\int_0^1 (4x-x^2)\,dx$.

✓ $x^2=4x$ at $x=0$ and $x=4$: $\int_0^4 (4x-x^2)\,dx = \frac{32}{3}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

2024 AB6(a) asks for the area of a region between two curves (top minus bottom, the ends where they meet).

8.4 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Area between two curves	Ends where they meet, test one x for the top, then $\int_a^b (\text{top} - \text{bottom})\,dx$.	E01, E02 · Practice 1, 3, 4, 6
Set up the integral	Write the integral with its ends and dx; evaluate only when asked.	Practice 2, 5

Example 01

Video · E01

R is the region between $y = x^2 - 4x$ and $y = -2x$.
(a) Find the area of R.
(b) Write an integral for the area of the part of R left of $x = 1$. Then find its value.

> The curves meet at $x = 0$ and $x = 2$. Test $x = 1$ for the top.

Example 02

Video · E02

What is the area of the region between $y = 2\sqrt{x}$ and $y = x$?

A. $\frac{8}{3}$
B. $-\frac{8}{3}$
C. $\frac{4}{3}$
D. $\frac{16}{3}$

> Where do $2\sqrt{x}$ and x meet? Test one x between.

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.4 / Example answers

Finding the Area Between Curves Expressed as Functions of x

E01 (a) $\frac{4}{3}$; (b) $\int_0^1 (2x - x^2)\,dx = \frac{2}{3}$

E02 (A)