8.3 / Practice workbook

Using Accumulation Functions and Definite Integrals in Applied Contexts

Practice

Problems 1–7 / Show your reasoning

1 Snow falls at a rate of $R(t)$ inches per hour, t hours after midnight. What does $\int_2^5 R(t)\,dt$ represent?

A. the inches of snow that fall from $t = 2$ to $t = 5$ hours

B. the rate of snowfall at $t = 5$

C. the change in the snowfall rate from $t = 2$ to $t = 5$

D. the inches of snow on the ground at $t = 5$

2 A tank holds 100 gallons of oil at $t = 0$. Oil leaks out at $L(t) = 4 + t$ gallons per hour. How much oil is in the tank at $t = 6$?

3 People enter a store at $E(t) = 40 - 4t$ and leave at $L(t) = 6t$ people per hour, $0 \le t \le 8$. There are 20 people at $t = 0$. When is the number of people greatest, and how many are there then?

4 $P'(t)$ is the rate of change of a town's population, in people per year. Explain the meaning of $\int_0^{10} P'(t)\,dt = 1200$.

8.3 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

Sand is added to a pile at a rate of 6 tons per hour and removed at a rate of $3\sqrt{t}$ tons per hour, for $0 \le t \le 9$. At $t = 0$ the pile has 40 tons of sand.

(a) How many tons of sand are in the pile at $t = 9$?

(b) At what time is the amount of sand in the pile greatest? Justify your answer.

(c) Is the amount of sand increasing or decreasing at $t = 6$? Give a reason.

8.3 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

A pump removes water from a tank at a rate of $R(t) = \dfrac{12}{t+1}$ liters per minute.

(a) How much water is removed from $t = 0$ to $t = e - 1$ minutes?

(b) Find the time T at which a total of 24 liters has been removed since $t = 0$.

8.3 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

Water flows into a tank at a rate of $R(t)$ gallons per hour, t hours after the tank starts to fill. Selected values of $R(t)$ are given in the table. At $t = 0$, the tank holds 40 gallons of water.

t	0	1	3	6
$R(t)$	10	14	12	8

(a) Use a trapezoidal sum with the three subintervals in the table to approximate $\int_0^6 R(t)\,dt$. Using correct units, explain the meaning of $\int_0^6 R(t)\,dt$ in the context of the problem.

(b) Use your answer from part (a) to estimate the amount of water in the tank at time $t = 6$.

8.3 / Answer key

Using Accumulation Functions and Definite Integrals in Applied Contexts

1 (A)

2 58 gallons

3 (a) $t = 4$ hours; (b) 100 people (ends: 20 and 20)

4 The population of the town increases by 1200 people from $t = 0$ to $t = 10$ years.

5 (a) 40 tons; (b) $t = 4$: $6 - 3\sqrt{t}$ changes from $+$ to $-$; $S(0) = 40$, $S(4) = 48$, $S(9) = 40$; (c) decreasing: $6 - 3\sqrt{6} < 0$

6 (a) 12 liters; (b) $T = e^2 - 1$ minutes

7 (a) 68; about 68 gallons flow into the tank from $t = 0$ to $t = 6$ hours; (b) 108 gallons