Lecture Notes · Unit 8 · Lecture 3 AP CED 8.3

Accumulation in context

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO: START + CHANGE

Rate In, Rate Out AP Calculus 8.3

20 gal at $t = 0$

in 2t, out 6 gal/min, 5 min

How much at $t = 5$? When lowest?

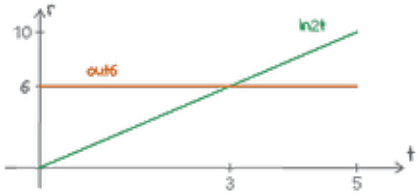
Gut: $\int$ (in − out), done?

Today: start + change, meaning in words, lowest or highest

IDEA 1: RATE IN MINUS RATE OUT

Rate in − rate out

1 graph: in 2t, out 6 (gal/min)



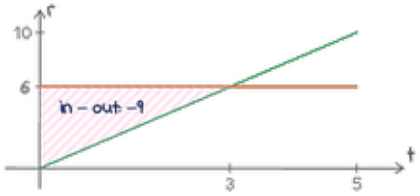
1. In − out

gallons in the pool at $t \rightarrow A(t)$

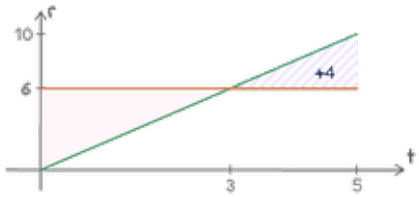
$A' = \text{in} - \text{out} = 2t - 6$

$\int_0^5 A'(t)\,dt = A(5) - A(0) = \text{change}$

$0 < t < 3$, out wins $\rightarrow -\frac{1}{2} \cdot 3 \cdot 6 = -9$



$3 < t < 5$, in wins $\rightarrow \frac{1}{2} \cdot 2 \cdot 4 = 4$



IDEA 2: ADD THE START, SAY WHAT IT MEANS

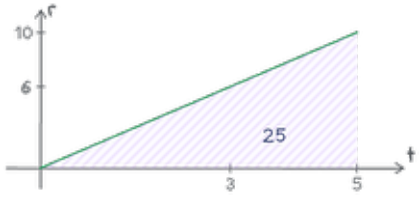
Add the start · say what it means

$A(5) = 20 + \int_0^5 (2t - 6)\,dt = 20 - 9 + 4 = 15$

$\int \rightarrow$ change only $\rightarrow$ add start

2. Say what it means

Inflow: $\int_0^5 2t\,dt = \frac{1}{2} \cdot 5 \cdot 10 = 25$



25 gallons flow in, $t = 0$ to $t = 5$ min

Not gal/min $\rightarrow$ gal/min × min = gal

AP 2023 Q1(a): amount, units, interval

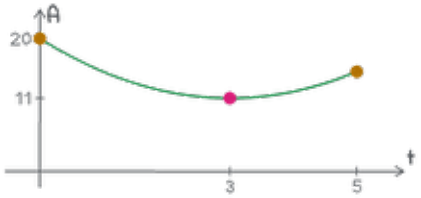
IDEA 3: THE LOWEST AMOUNT

Lowest or highest: candidates

3. The lowest amount

$A' = 2t - 6 = 0 \rightarrow t = 3$

$A(0) = 20$, $A(3) = 20 - 9 = 11$, $A(5) = 15$



11 gal at t = 3 → compare ends

IDEA 4: TABLES, SIGNS, SETUPS

Read what you're given

4. what the problem gives you

Table of A(t) → change = A(b) − A(a)

Rate table → 6.2: Riemann sum ≈ ∫

out given as +6 → subtract it

A = 16, when? → $20 + \int_0^T (2t - 6)\,dt = 16$

Rate in t only → just integrate

PROBLEM 1 OF 2

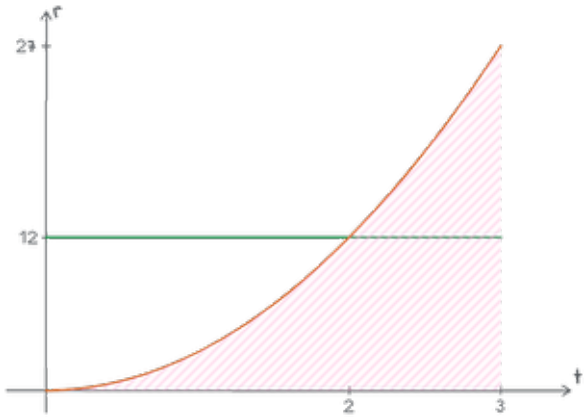
Water in a tank

A tank holds 50 gallons at $t = 0$. Water is pumped in at 12 gal per hour and leaks out at $3t^2$ gal per hour, $0 \le t \le 3$.

(a) Explain the meaning of $\int_0^3 3t^2\,dt$ in the context of the problem.

(b) How much water is in the tank at $t = 3$?

(c) When is the amount greatest? Justify your answer.



Solution (a) amount + interval, (b) start + integral

Plan: (a) amount, interval (b) start + ∫

(a)

$$\int_0^3 3t^2\,dt = [t^3]_0^3 = 27$$

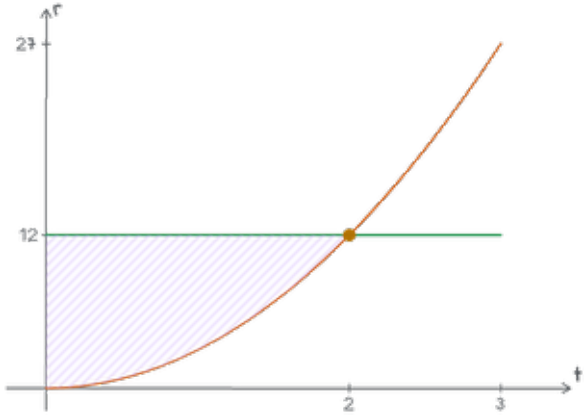
27 gallons leak out, t = 0 to t = 3 hr

(b)

$$A(3) = 50 + \int_0^3 (12 - 3t^2)\,dt$$

$$= 50 + 12 \cdot 3 - 27 = 50 + 36 - 27 = 59$$

Solution (c) A' = 0, then every candidate



(c)

A′ = 0 → compare

$$A'(t) = 12 - 3t^2 = 0 \rightarrow t = 2$$

A(0) = 50, A(2) = 50 + 24 − 8 = 66, A(3) = 59 (b)

Greatest at t = 2 → 66 gal → largest candidate

Workbook: start + in − out

Careful → where points are lost

✗ Common mistake

27 gal per hour leak out

A′(2) = 0 → max

✓ Do this

27 gal, t = 0 to 3

Compare A(0), A(2), A(3)

PROBLEM 2 OF 2

Which expression

People enter a park at $E(t)$ people per hour and leave at $L(t)$ people per hour. At $t = 0$, N people are in the park. Which of the following gives the number of people in the park at time T?

(A) $N + \int_0^T (E(t) - L(t))dt$

(B) $E(T) - L(T)$

(C) $\int_0^T (E(t) - L(t))dt$

(D) $N + E(T) - L(T)$

Solution

End = start + integral of the rate; each wrong choice in one line

Plan: start + ∫(in − out) dt

(B) E(T) − L(T) → people per hour, one instant

(C) change only → start N missing

(D) people + people per hour

So (A) → $N + \int_0^T (E - L)\,dt$

Workbook: units

WRAP-UP

Pool: 15 gal, lowest 11

Amount = start + ∫(in − out) dt

∫ of a rate → amount, units, interval

Lowest or highest → A′ = 0, then the ends

Workbook → 8.4: 2 curves

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.