8.3 / Using Accumulation Functions and Definite Integrals in Applied Contexts

Watch the lesson

➤

Amount = start + the integral of (in - out), the meaning sentence, the least and the most

Learning goal

Explain what an integral of a rate means in context, find an amount from a start and rates in and out, and find when the amount is greatest or least.

Key ideas

01 The integral of a rate over $[a, b]$ is an **amount**: how much of the quantity is added (or removed) from $t = a$ to $t = b$, in the units of the quantity.

02 The meaning sentence names the amount, the units, and the **time interval**. It never says "rate".

03 Amount at time $T =$ **start** $+ \int_0^T (\text{in} - \text{out})\,dt$. A rate of leaving is given as a positive number, so it is subtracted.

04 Greatest or least amount: in $-$ out changes sign at the candidates; compare the amount there and at both **endpoints**.

05 A table of a rate is added up with a Riemann or trapezoidal **sum**. A table of the amount itself gives the change by subtraction.

Remember

rate in, rate out
How fast the quantity comes in and goes out, per unit of time.

net change
$\int_a^b (\text{in} - \text{out})\,dt$: the amount gained minus the amount lost.

candidate
A time where the amount could be greatest or least: a sign change of in $-$ out, or an endpoint.

Class notes

8.3 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the meaning with the amount, units, and interval: ”27 gallons of water leak out of the tank from $t = 0$ to $t = 3$ hours.”	Do not describe an integral of a rate as a rate (no ”gallons per hour”).
Show the candidates in a short list with their amounts, then name the greatest.	Do not leave out the starting amount, or the endpoints when you look for the greatest amount.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Start with what was there

Code V2

✗ Water at $t = 3$: $\int_0^3 (12 - 3t^2)\,dt = 9$ gallons.

✓ $50 + \int_0^3 (12 - 3t^2)\,dt = 50 + 9 = 59$ gallons.

02 Check the endpoints too

Code V5

✗ In = out at $t = 2$, so the amount is greatest at $t = 2$.

✓ Compare $A(0) = 50$, $A(2) = 66$, $A(3) = 59$: greatest at $t = 2$, with a reason.

Real AP questions on this topic Free on AP Central. Try them after the practice.

2023 AB1(a) asks what a definite integral of a rate of flow means in context, and to approximate it from an unequally spaced table.

8.3 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
The meaning of an integral	An amount with its units, and the time interval. Never a rate.	Practice 1, 4
Amount from rates in and out	Start, plus the integral of in minus out, with the dt.	E01, E02 · Practice 2, 6, 7
Least or most	In = out gives the candidates; compare them with the start and the end.	Practice 3, 5

Example 01

Video · E01

A tank holds 50 gallons at $t = 0$. Water is pumped in at 12 gal per hour and leaks out at $3t^2$ gal per hour, $0 \le t \le 3$.
(a) Explain the meaning of $\int_0^3 3t^2\,dt$ in context.
(b) How much water is in the tank at $t = 3$?
(c) When is the amount greatest? Justify.

> The integral of the leak rate is an amount of water. For the greatest amount, compare the candidate with both ends.

Example 02

Video · E02

People enter a park at $E(t)$ people per hour and leave at $L(t)$ people per hour. At $t = 0$, N people are in the park. Which gives the number in the park at time T?

A. $N + \int_0^T (E(t) - L(t))\,dt$
B. $E(T) - L(T)$
C. $\int_0^T (E(t) - L(t))\,dt$
D. $N + E(T) - L(T)$

> People in the park = people at the start + people who came in − people who left.

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.3 / Example answers

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E01 (a) 27 gallons of water leak out of the tank from $t = 0$ to $t = 3$ hours; (b) 59 gallons; (c) at $t = 2$: 66 gallons ($A(0) = 50, A(3) = 59$)

E02 (A)