8.2 / Practice workbook

Connecting Position, Velocity, and Acceleration Using Integrals

Practice

Problems 1–7 / Show your reasoning

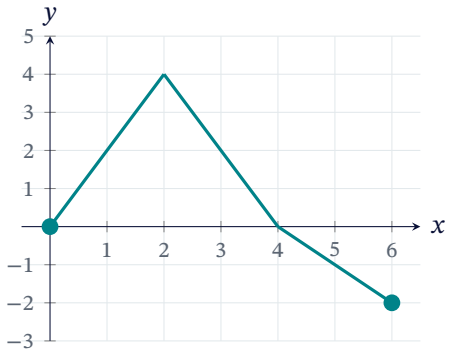
1 A particle moves along the x-axis with velocity $v(t) = 4 - t^2$ and $x(0) = 1$. Find $x(3)$.

2 A particle has acceleration $a(t) = 6t$ and $v(0) = -3$. What is $v(2)$?

A. 9
B. 12
C. 15
D. 6

3 For the particle in Problem 1, $v(t) = 4 - t^2$ on $0 \le t \le 3$, find the total distance traveled.

4 The graph of v on $[0, 6]$ is made of segments $(0, 0) \to (2, 4) \to (4, 0) \to (6, -2)$, and $x(0) = -1$. Find $x(6)$, and the particle's farthest position to the right.



8.2 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

A particle moves along the x-axis with velocity $v(t) = 2\cos t$ for $0 \le t \le \frac{3\pi}{2}$, and $x(0) = 1$.

(a) Find $x\left(\frac{3\pi}{2}\right)$.

(b) Find the total distance traveled by the particle over $0 \le t \le \frac{3\pi}{2}$.

(c) Find the particle's farthest position to the left. Justify your answer.

8.2 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

A particle moves along the x-axis with acceleration $a(t) = 6t - 6$, $v(0) = 3$, and $x(0) = 0$.

(a) Find $v(t)$.

(b) Does the particle ever change direction? Explain.

(c) Find $x(2)$.

8.2 / AP-style free response

Problem 7 ➤ / *Show the work and the reasons a grader looks for.*

Particle P moves along the x-axis with velocity $v_P(t) = 2t - 4$, and $x_P(0) = 3$. Particle Q moves along the x-axis with position $x_Q(t) = t + 1$. Both move for $0 \le t \le 4$.

(a) Find the position of particle P at time $t = 4$.

(b) Find the distance between the two particles at time $t = 4$.

(c) Find the total distance traveled by particle P over $0 \le t \le 4$. Explain why it is not $|x_P(4) - x_P(0)|$.

8.2 / Answer key

Connecting Position, Velocity, and Acceleration Using Integrals

1 $x(3) = 4$

2 (A)

3 $\frac{23}{3}$

4 (a) $x(6) = 5$; (b) farthest right $x = 7$, at $t = 4$

5 (a) -1; (b) 6; (c) $x = -1$ at $t = \frac{3\pi}{2}$ (candidates $x(0) = 1$, $x\left(\frac{\pi}{2}\right) = 3$, $x\left(\frac{3\pi}{2}\right) = -1$)

6 (a) $v(t) = 3(t-1)^2$; (b) no: $v(t) = 3(t-1)^2 \geq 0$; $v(1) = 0$, but v does not change sign; (c) $x(2) = 2$

7 (a) 3; (b) 2; (c) 8; P turns around at $t = 2$, and $|x_P(4) - x_P(0)| = 0$ is only the distance between its two positions