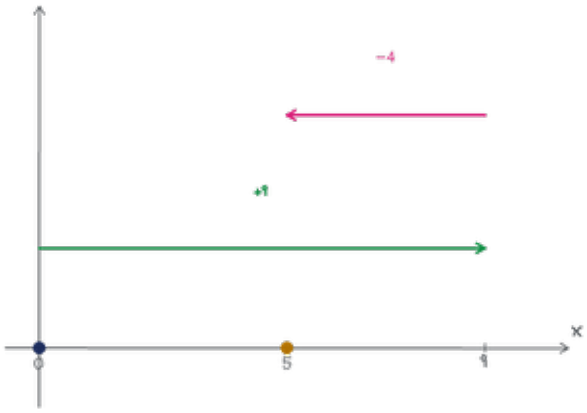
**Lecture Notes · Unit 8 · Lecture 2** AP CED 8.2

# Motion with integrals

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

## INTRO: GPS OR ODOMETER?

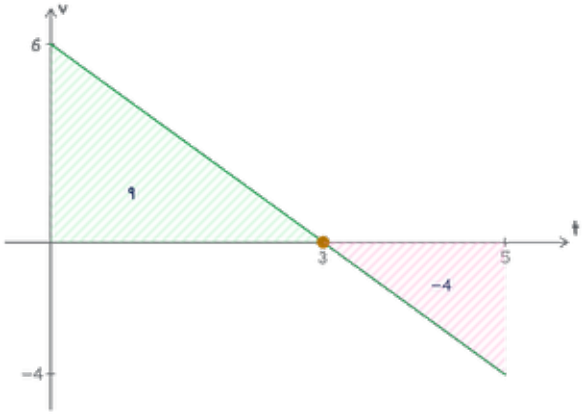
Position, Displacement & Total Distance
AP Calculus 8.2




9 m forward → 4 m back
GPS: where? 9 − 4 = 5 m
how far? 9 + 4 = 13 m
Today → both from only v(t); v from a(t)

## IDEA 1: AREA IS THE CHANGE IN POSITION

Area = change in position




only v → $v(t) = 6 - 2t$ m/s, $0 \le t \le 5$

$x' = v \rightarrow \int_0^5 v(t)\,dt = x(5) - x(0)$

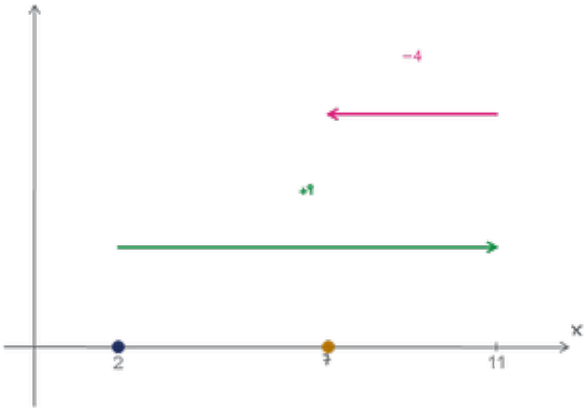
0 to 3 → $\frac{1}{2} \cdot 3 \cdot 6 = 9$, forward

3 to 5 → $-\frac{1}{2} \cdot 2 \cdot 4 = -4$

$\int_0^5 v(t)\,dt$ → net position change

## DISPLACEMENT AND POSITION

9 − 4 = 5 → displacement



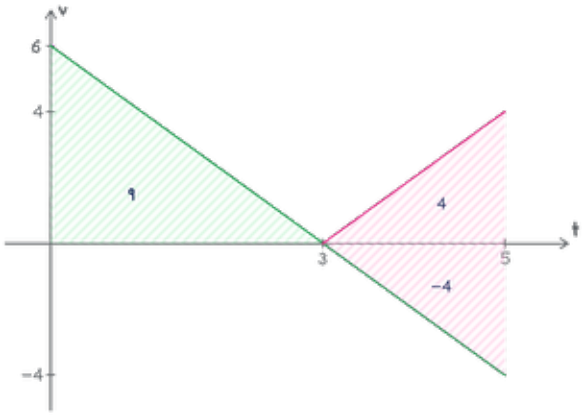

$x(5) = x(0) + \int_0^5 v(t)\,dt = 2 + 9 - 4 = 7$

$x(b) = x(a) + \int_a^b v(t)\,dt$

Given x(2) → $x(5) = x(2) + \int_2^5 v(t)\,dt$

## IDEA 2: FLIP FOR THE DISTANCE

The odometer: total distance




Odometer → speed = $|v|$

Flip pink up → $\int_0^5 |v(t)|\,dt = 9 + 4 = 13$

Total distance = $\int_a^b |v(t)|\,dt$

$\left|\int_0^5 v(t)\,dt\right| = 5$ → gap, not 13

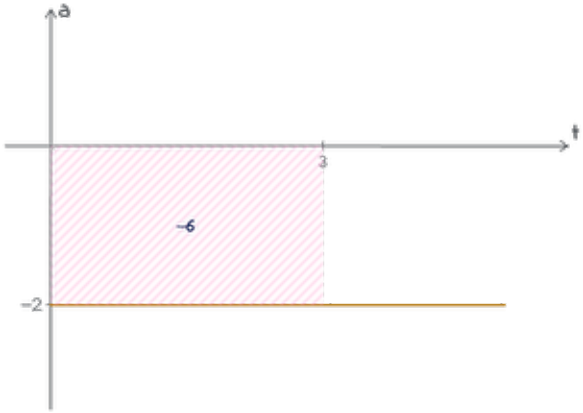
AP: gap between 2 positions → $|x(b) - x(a)|$

By hand → split where v changes sign

Calc: $\int_a^b |v(t)|\,dt$

## IDEA 3: ONE LAYER DOWN

One layer down: a to v




$a(t) = v'(t) = -2$

on $[0, 3]$ → $-2 \cdot 3 = -6$

$v(3) = v(0) + \int_0^3 a(t)\,dt = 6 - 6 = 0$

Matches → stops at $t = 3$

End = start + $\int$ rate $dt$

## PROBLEM 1 OF 2

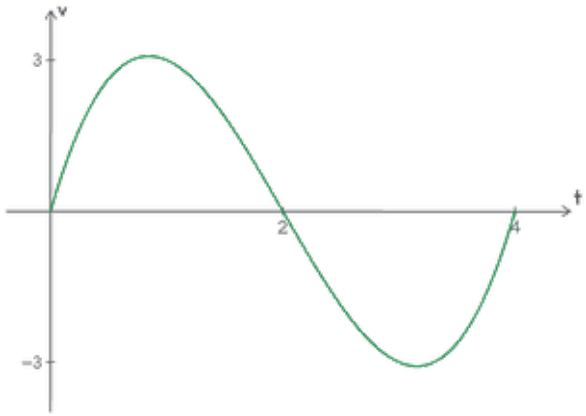
Position and distance

A particle moves along the $x$-axis with velocity $v(t) = t^3 - 6t^2 + 8t$, $0 \le t \le 4$, and $x(0) = 1$.

(a) Find the position at $t = 4$.

(b) Find the total distance traveled.

(c) Find the particle's farthest position to the right.




**Solution** Position = start + the integral of v

Plan: (a) start + $\int_0^4 v(t)\,dt$

(a)

$$x(4) = 1 + \int_0^4 (t^3 - 6t^2 + 8t)\,dt$$

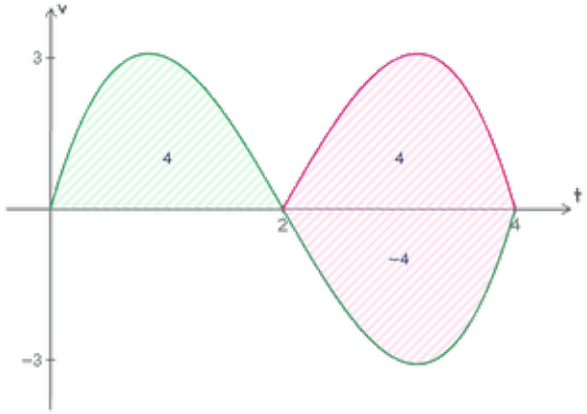
$$= 1 + \left[\frac{t^4}{4} - 2t^3 + 4t^2\right]_0^4$$

$$= 1 + (64 - 128 + 64) = 1$$

→ back at the start

Part (b): distance, not position

**Solution** Distance = the integral of |v|: split, flip, add




(b) v changes sign → split, flip

(b)

$v = t(t-2)(t-4)$

0, 4: ends → t = 2 sign change

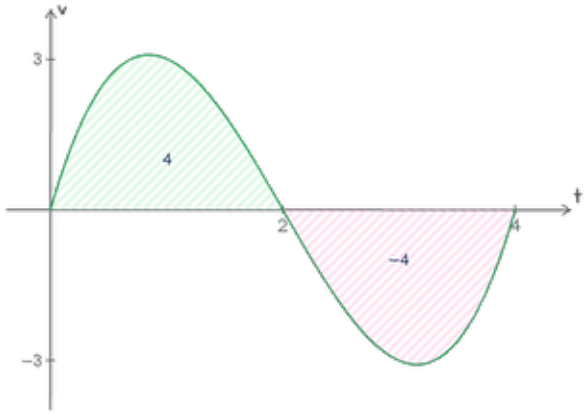
$$\int_0^2 v(t)\,dt = 4 - 16 + 16 = 4$$

$$\int_2^4 v(t)\,dt = 0 - 4 = -4$$

$$\int_0^4 |v(t)|\,dt = 4 + 4 = 8$$

Back at start, still 8 → don't add x(0)

**Solution** Farthest right: candidates, then start + area




(c)

+ → − t = 2 → t = 0, 2, 4

$x(0) = 1,\ x(2) = 1 + 4 = 5,\ x(4) = 1$

Farthest right x = 5 t = 2

Workbook: farthest left → 2 particles

$|x_P(t) - x_Q(t)|$

Careful → two hurried lines

X Common mistake

$$x(4) = \int_0^4 v(t) + 1$$

$$\text{Distance} = \left|\int_0^4 v(t)\,dt\right| = 0$$

✓ Do this

$$x(4) = 1 + \int_0^4 v(t)\,dt$$

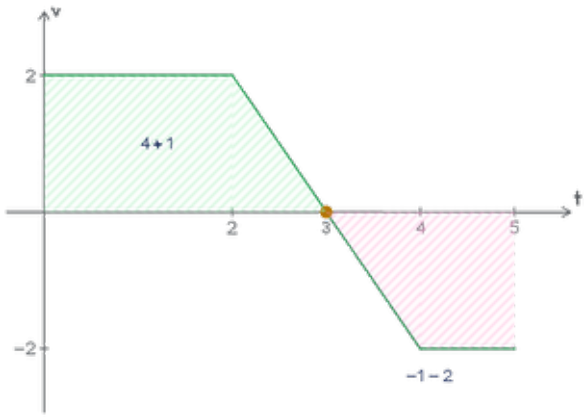
$$\text{Dist.} = \int_0^4 |v(t)|\,dt = 4 + 4 = 8$$

**PROBLEM 2 OF 2**

Position and distance from a graph

The graph of the velocity $v$ of a particle consists of line segments:
$(0, 2) \to (2, 2) \to (4, -2) \to (5, -2)$, and $x(0) = 5$.
Which of the following gives the position at $t = 5$ and the total distance traveled?

(A) 7; 8
(B) 7; 2
(C) 13; 8
(D) 3; 8




**Solution**

Signed areas for the position, flipped areas for the distance

Signed → x flipped → distance

Midpoint $t = 3$

Above: $2 \cdot 2 + \frac{1}{2} \cdot 1 \cdot 2 = 5$

Below: $\frac{1}{2} \cdot 1 \cdot 2 + 1 \cdot 2 = 3$

$x(5) = 5 + (5 - 3) = 7$

distance $= 5 + 3 = 8$

So (A)

(B) 2 = displacement (C) 13 = 8 + 5

workbook → signed x → flipped distance

**WRAP-UP**

where, and how far

Position = start $+ \int_a^b v(t)\,dt$

Distance $= \int_a^b |v(t)|\,dt$ → flip what's below

Gap $= |x(b) - x(a)| \neq$ total distance

1 layer down → $v(b) = v(a) + \int_a^b a(t)\,dt$

Exam → start first, dt at the end

workbook · next → 8.3 accumulation in context