8.2 / Connecting Position, Velocity, and Acceleration Using Integrals

Watch the lesson ➤

Position = start + the integral of v, total distance = the integral of |v|, velocity from acceleration

Learning goal

From a velocity, find a position, the displacement, and the total distance, and keep the three apart; find the farthest position from the candidates.

Key ideas

01 Position at time b = **start** + displacement: $x(b) = x(a) + \int_a^b v(t)\,dt$. The lower end is the given starting time.

02 **Displacement** $\int_a^b v(t)\,dt$ adds signed areas: moving left counts negative.

03 **Total distance** $\int_a^b |v(t)|\,dt$: split where v changes sign and add the sizes of the pieces (with a calculator, integrate $|v|$ at once).

04 The farthest right or left position: the **candidates** are the endpoints and the times where v changes sign; compare the positions there.

05 In the same way, velocity at b = starting velocity + $\int_a^b a(t)\,dt$. The distance between two particles at time t is $|x_P(t) - x_Q(t)|$.

Remember

displacement
Change in position, $\int_a^b v(t)\,dt$. It can be negative.

total distance
How far the particle actually travels: $\int_a^b |v(t)|\,dt$.

speed
$|v(t)|$, the size of the velocity.

Class notes

8.2 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the start in front: $x(4) = 1 + \int_0^4 v(t)\,dt$.	Do not give $\int_a^b v(t)\,dt$ as the total distance when v changes sign.
For total distance, show where you split: $\int_0^2 v\,dt - \int_2^4 v\,dt$, or $\int_0^4 \lvert v(t)\rvert\,dt$.	Do not give the position as the integral alone; the starting position is part of it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The start is part of the position

Code V2

✗ $v(t) = 4 - t^2$, $x(0) = 1$: $x(3) = \int_0^3 (4 - t^2)\,dt = 3$.

✓ $x(3) = 1 + \int_0^3 (4 - t^2)\,dt = 1 + 3 = 4$.

02 Distance does not cancel

Code V3

✗ Total distance on $[0, 3]$: $\int_0^3 (4 - t^2)\,dt = 3$.

✓ v changes sign at $t = 2$: $\frac{16}{3} + \frac{7}{3} = \frac{23}{3}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

2023 AB2 asks when a particle changes direction, the distance between its positions at two times, and the total distance traveled (the integral of the speed).

8.2 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Position and displacement	Start in front, plus the integral of v, with the dt.	E02 · Practice 1, 4
Total distance	Find where v changes sign, flip the parts below the axis, add.	E01 · Practice 3, 5, 7
Velocity from acceleration	$v(b) = v(a) + \int_a^b a(t)\,dt$, one layer down.	Practice 2, 6

Example 01

Video · E01

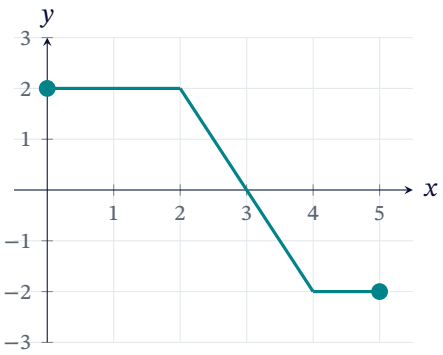
A particle moves along the x-axis with velocity $v(t) = t^3 - 6t^2 + 8t$, $0 \le t \le 4$, and $x(0) = 1$.
(a) Find the position at $t = 4$.
(b) Find the total distance traveled.
(c) Find the particle's farthest position to the right.

$v = t(t-2)(t-4)$ changes sign at $t = 2$. The candidates for the farthest right are $t = 0, 2, 4$.

Example 02

Video · E02

The velocity v of a particle is made of segments: $(0, 2) \to (2, 2) \to (4, -2) \to (5, -2)$, and $x(0) = 5$. Position at $t = 5$, and total distance?



A. 7; 8
B. 7; 2
C. 13; 8
D. 3; 8

> Position: start plus signed areas. Distance: every area counts as positive.

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.2 / Example answers

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E01 (a) $x(4) = 1$; (b) 8; (c) $x = 5$ at $t = 2$

E02 (A)