8.1 / Practice workbook

Finding the Average Value of a Function on an Interval

Practice

Problems 1–7 / Show your reasoning

1 Find the average value of $f(x) = 3x^2 - 2x$ on $[1, 3]$.

2 What is the average value of $g(x) = \cos x$ on $\left[0, \frac{\pi}{2}\right]$?

A. 0
B. $\frac{2}{\pi}$
C. 1
D. $\frac{\pi}{2}$

3 The graph of f on $[0, 4]$ is the upper half of the circle with center $(2, 0)$ and radius 2. Find the average value of f on $[0, 4]$.

4 Let $f(x) = x^2 + 1$. Find the value of c in $[0, 3]$ such that $f(c)$ equals the average value of f on $[0, 3]$.

8.1 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

f is continuous on $[1, 5]$ and $\int_1^5 f(x)\,dx = 12$.

(a) Find the average value of f on $[1, 5]$.

(b) Find the average value of $2f(x) + 1$ on $[1, 5]$.

(c) Must there be a value c in $[1, 5]$ such that $f(c) = 3$? Explain.

6 The average value of $f(x) = x^3$ on $[0, b]$ is 16.
Find $b > 0$.

8.1 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

The temperature $T(t)$, in degrees Fahrenheit, is recorded at the times t, in hours after noon, given in the table.

t (hours)	0	2	3	6
$T(t)$ (°F)	48	56	60	54

(a) Use a trapezoidal sum with the three subintervals to estimate the average value of T over $0 \le t \le 6$. Include units.

(b) Find the average rate of change of T over the same interval. Include units.

8.1 / Answer key

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1 9

2 (B)

3 $\frac{\pi}{2}$

4 $c = \sqrt{3}$

5 (a) 3; (b) 7; (c) yes: f is continuous on $[1, 5]$, so it reaches its average value 3 at least once

6 $b = 4$

7 (a) 55.5 degrees Fahrenheit; (b) 1 degree Fahrenheit per hour