Lecture Notes · Unit 8 · Lecture 1 AP CED 8.1

Average value of a function

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INTRO: ONE STEADY SPEED

Average Value of a Function AP Calculus 8.1

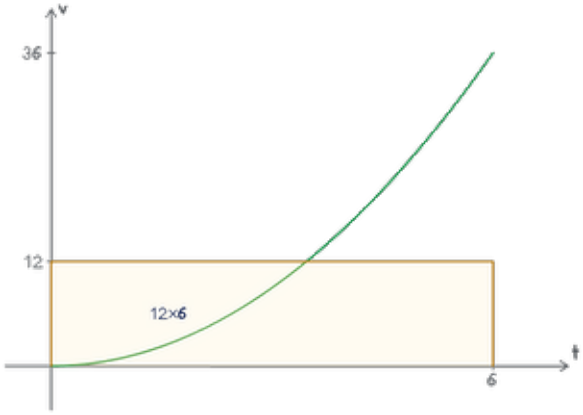
$v(t) = t^2$, $0 \le t \le 6$ → 0 to 36 ft/s

1 steady speed → same distance?

Halfway? 18

Today: real answer, where v hits it, 2 averages

ONE STEADY HEIGHT



Distance $= \int_0^6 t^2\,dt = \frac{6^3}{3} = 72$

1 steady s: rectangle $s \cdot 6 = 72$ → $s = 12$

18? No → slow for longer

$s = \frac{1}{6-0}\int_0^6 v(t)\,dt = \frac{72}{6} = 12$

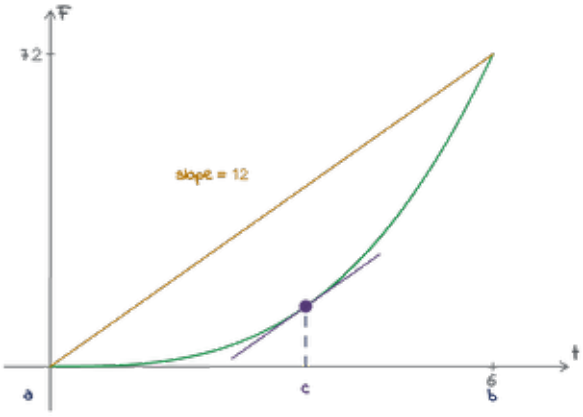
$0 \to a$, $6 \to b$, $v \to$ continuous f

$\frac{1}{b-a}\int_a^b f(x)\,dx$

→ average value of f on $[a,b]$

12 ft/s: same units as v

WHERE F HITS ITS AVERAGE



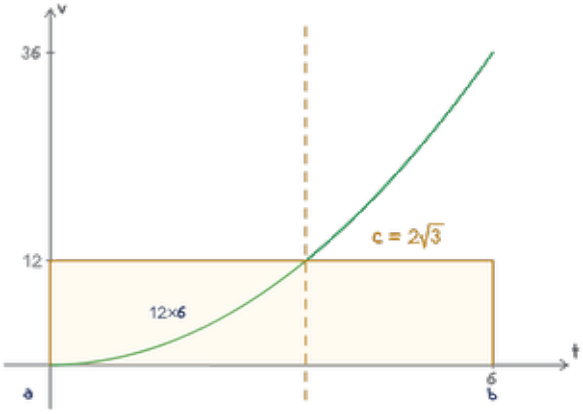
$F' = f \to \int_a^b f(x)\,dx = F(b) - F(a)$

$\to$ Avg $= \frac{F(b)-F(a)}{b-a}$ → slope of F

MVT: F differentiable → continuous

Car: $F = \frac{t^3}{3}$, slope $\frac{72}{6} = 12$ → tangent at c

$f(c) = F'(c) = \frac{F(b)-F(a)}{b-a}$



Same 12 → height of v

If f continuous on $[a,b]$

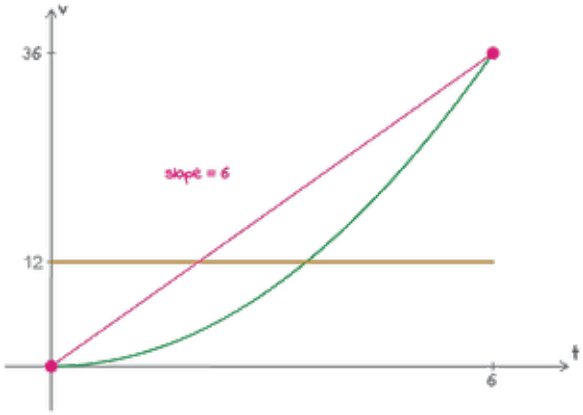
→ at least one c in (a,b):

$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

Mean Value Theorem for Integrals

$c^2 = 12,\ c > 0 \rightarrow c = 2\sqrt{3} = \sqrt{12} < \sqrt{36} = 6$

AVERAGE VALUE VS. AVERAGE RATE OF CHANGE



Average value of v: $\frac{72-0}{6-0} = 12$ = slope of F

Average rate of change of v: $\frac{36-0}{6-0} = 6$

6 ft/s per second, on average

Average value of f → slope of F → 12

Avg. rate of f → slope of f

Avg speed, avg flow rate → $\frac{1}{b-a}\int_a^b f(x)\,dx$

PROBLEM 1 OF 2

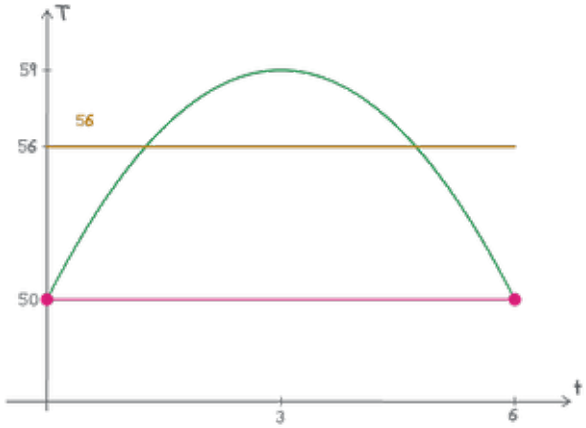
Average temperature

The temperature, in °F, t hours after 9 a.m. is $T(t) = 50 + 6t - t^2$ for $0 \le t \le 6$.

(a) Find the average temperature over $0 \le t \le 6$.

(b) Find the average rate of change of T over $0 \le t \le 6$.

(c) Find all values of t for which $T(t)$ equals the average temperature.



Solution (a) a height, (b) a slope

Plan: (a) height → integral ÷6 (b) slope → two ends

(a)

$$\frac{1}{6}\int_0^6 T(t)\,dt = \frac{1}{6}\left[50t + 3t^2 - \frac{t^3}{3}\right]_0^6$$

$$= \frac{300 + 108 - 72}{6} = \frac{336}{6} = 56\,^\circ F$$

56 °F = average (a height)

(b)

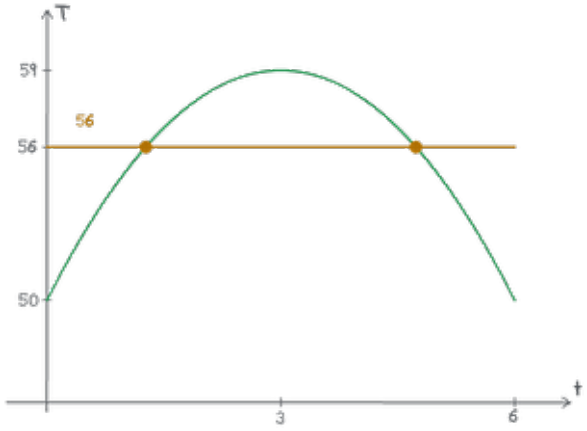
$$\frac{T(6) - T(0)}{6-0} = \frac{50-50}{6} = 0$$

0 °F per hour → flat (50 → 50)

Average of the ends $\frac{50+50}{2} = 50 \neq 56$

Part (c): your turn

Solution (c) Where T hits the average



(c)

Plan: $T = 56 \rightarrow$ solve

$50 + 6t - t^2 = 56 \rightarrow t^2 - 6t + 6 = 0$

$t = \frac{6 \pm \sqrt{36 - 24}}{2} = \frac{6 \pm \sqrt{12}}{2} = 3 \pm \sqrt{3}$

→ two yellow dots

$t = 3 \pm \sqrt{3} \rightarrow \sqrt{3} < 3$, both inside

Workbook: height → where f hits it

TWO TRAPS IN PART (A)

Wrong: $\int_0^6 T(t)\,dt = \frac{336}{6} = 56 \rightarrow$ first = false

Wrong: avg temp = $\frac{T(6) - T(0)}{6} \rightarrow$ slope of T

Right: $\frac{1}{6}\int_0^6 T(t)\,dt = 56$

AP 2023 Q1(c): a false = → 1 of 2 points

Every = true; $\frac{1}{b-a}$ first

PROBLEM 2 OF 2

Average value from a graph

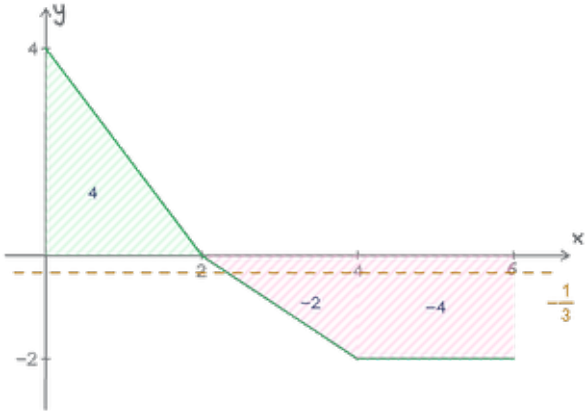
The graph of f on $[0, 6]$ is made of segments:
$(0, 4) \rightarrow (2, 0) \rightarrow (4, -2) \rightarrow (6, -2)$.
What is the average value of f on $[0, 6]$?

(A) -2

(B) $-\frac{1}{3}$

(C) $\frac{1}{3}$

(D) $\frac{5}{3}$



Solution Signed area, then the width

Plan: signed area ÷ width 6

0 to 2: $\frac{1}{2} \cdot 2 \cdot 4 = 4$

2 to 4: $-\frac{1}{2} \cdot 2 \cdot 2 = -2$

4 to 6: $-2 \cdot 2 = -4$

$\frac{1}{6}\int_0^6 f(x)\,dx = \frac{1}{6}(4 - 2 - 4) = -\frac{1}{3}$

So (B): $-\frac{1}{3}$

(A) no ÷6 (D) below as +

Workbook: graphs, tables → Riemann sum ÷ width

WRAP-UP



one steady height

Average value = $\frac{1}{b-a}\int_a^b f(x)\,dx$: a height

f continuous → at least one c: f(c) = average

value → slope of F (∫), rate of change → slope of f (ends)

Every = true

Workbook · Next → 8.2: position from velocity

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.