8.1 / Finding the Average Value of a Function on an Interval

Watch the lesson ➤

The average value as a steady height, where f hits it, and height versus slope

> **Learning goal**
> Find the average value of a function on an interval as a steady height, find where the function reaches it, and keep it apart from the average rate of change.

Key ideas

01 The **average value** of f on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$: the steady height that gives the same area over the same width.

02 Its units are the units of f. From a graph, use signed areas. From a table, estimate the integral with a Riemann or **trapezoidal** sum, then divide by the width.

03 If $F' = f$, then $\int_a^b f(x)\,dx = F(b) - F(a)$. So the average value of f is the **average rate of change** of F.

04 If f is **continuous** on $[a, b]$, the Mean Value Theorem for F gives at least one c in (a, b) with $f(c)$ equal to the average value.

05 The average rate of change of f itself is $\frac{f(b)-f(a)}{b-a}$, a **slope**. It is not the average value, and neither is $\frac{f(a)+f(b)}{2}$.

Remember

average value
The integral divided by the width of the interval: a height.

average rate of change
The change in f divided by the change in x: a slope.

Mean Value Theorem for integrals
A continuous f reaches its average value at least once on the interval.

Class notes

8.1 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Show the setup with the integral, then the units of f: $\frac{1}{6-0}\int_0^6 T(t)\,dt = 56$ degrees Fahrenheit.	Do not use $\frac{f(a)+f(b)}{2}$ for an average value.
To find where f reaches its average, write the equation $f(c) =$ (average) and keep only the c in the interval.	Do not answer an average value with a slope, or an average rate of change with an integral.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 A height, not a slope

Code V1

✗ Average temperature over $0 \le t \le 6$: $\frac{T(6)-T(0)}{6} = 0$.

✓ Average temperature: $\frac{1}{6}\int_0^6 T(t)\,dt = 56$ degrees. The 0 is the average rate of change.

02 Not the average of the two ends

Code V1

✗ Average value of $3x^2 - 2x$ on $[1, 3]$: $\frac{1+21}{2} = 11$.

✓ $\frac{1}{2}\int_1^3 (3x^2 - 2x)\,dx = \frac{1}{2}(18) = 9$.

Real AP questions on this topic Free on AP Central. Try them after the practice.
2023 AB1(c) asks for the average rate of flow of a given rate of flow: the average value of that rate, an integral divided by the length of the time interval.

8.1 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Find an average value	The area (the integral, or shapes) divided by the width: $\frac{1}{b-a}\int_a^b f(x)\,dx$, in one true line.	E02 · Practice 1, 2, 3, 6, 7
Where f hits its average	Average value first, then solve $f(c) =$ average for c in $[a, b]$ (f continuous).	E01 · Practice 4, 5

Example 01

Video · E01

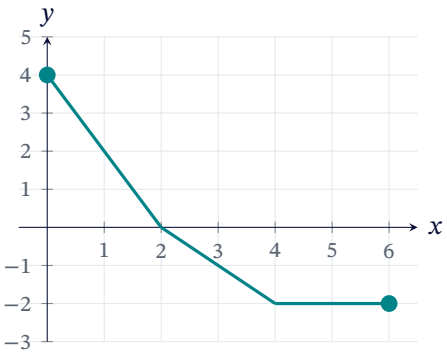
The temperature, in °F, t hours after 9 a.m. is $T(t) = 50 + 6t - t^2$ for $0 \le t \le 6$.
(a) Find the average temperature over $0 \le t \le 6$.
(b) Find the average rate of change of T over $0 \le t \le 6$.
(c) Find all t with $T(t)$ equal to the average temperature.

> The average temperature is a height: the integral over the width. The average rate of change is a slope.

Example 02

Video · E02

The graph of f on $[0,6]$ is made of segments: $(0,4) \to (2,0) \to (4,-2) \to (6,-2)$.
What is the average value of f on $[0,6]$?



A. -2
B. $-\frac{1}{3}$
C. $\frac{1}{3}$
D. $\frac{5}{3}$

> Signed areas: above the axis counts plus, below counts minus. Then divide by the width 6.

VIDEO COMPLETE Check your example answers at the end of this booklet.

8.1 / Example answers

Finding the Average Value of a Function on an Interval

E01 (a) 56 degrees Fahrenheit; (b) 0 degrees Fahrenheit per hour; (c) $t = 3 - \sqrt{3}$ and $t = 3 + \sqrt{3}$

E02 (B)