7.8 / Practice workbook

Exponential Models with Differential Equations

Practice

Problems 1–7 / Show your reasoning

1 A population P grows at a rate proportional to its size. $P(0) = 200$ and $P(2) = 1800$. Find k and write $P(t)$.

2 If $\frac{dy}{dt} = ky$ with $k = \frac{\ln 2}{5}$ and $y(0) > 0$, which of the following is true?

A. y doubles every 5 units of time.

B. y doubles every $\frac{\ln 2}{5}$ units of time.

C. y increases by 2 every 5 units of time.

D. y is cut in half every 5 units of time.

7.8 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

A substance decays at a rate proportional to the amount A present. Its half-life is 10 days, and $A(0) = 400$ grams.

(a) How much is left after 30 days?

(b) Find $\frac{dA}{dt}$ when $A = 100$. Explain the meaning of your answer in the context of the problem.

4 An amount grows at a rate proportional to itself and triples every 2 years. After how many years is it 27 times its starting size?

A. 4
B. 6
C. 13.5
D. 18

7.8 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

The number of bacteria B in a culture satisfies $\frac{dB}{dt} = kB$, with t in hours. $B(2) = 400$ and $B(5) = 3200$.

(a) Find k.

(b) Find $B(0)$.

(c) When is $B = 12800$?

6 A lake has 1000 fish, and the number P grows at a rate proportional to P. At the start, P is increasing at 50 fish per year. How many fish are there after 20 years?

A. $1000e$
B. 2000
C. $1000e^{20}$
D. 1050

7.8 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

A quantity y satisfies $\frac{dy}{dt} = 3(y - 10)$, with $y(0) = 12$.

(a) A student writes $y = 12e^{3t}$. Explain why this is not the solution.

(b) Find the solution. (Hint: let $u = y - 10$.)

7.8 / Answer key

Exponential Models with Differential Equations

1 $k = \ln 3$; $P(t) = 200e^{(\ln 3)t} = 200 \cdot 3^t$

2 (A)

3 (a) 50 grams (three half-lives); (b) $-10 \ln 2$: when $A = 100$ g, the amount is decreasing at $10 \ln 2$ grams per day

4 (B)

5 (a) $k = \ln 2$; (b) $B(0) = 100$; (c) $t = 7$ hours

6 (A)

7 (a) the rate is proportional to $y - 10$, not to y: $12e^{3t}$ gives $\frac{dy}{dt} = 3y \neq 3(y - 10)$; (b) $y = 10 + 2e^{3t}$