7.7 / Practice workbook

Finding Particular Solutions Using Initial Conditions and Separation of Variables

Practice

Problems 1–9 / Show your reasoning

1 Find the particular solution of $\frac{dy}{dx} = \frac{x}{y}$ with the initial condition $y(0) = -3$.

2 Which is the solution of $\frac{dy}{dx} = \frac{3y}{x}$, $x > 0$, with $y(1) = 2$?

A. $y = 2x^3$
B. $y = x^3 + 1$
C. $y = e^2x^3$
D. $y = 3\ln x + 2$

3 Find the particular solution $y = f(x)$ of $\frac{dy}{dx} = \frac{2x+2}{3y^2}$ with $f(1) = 2$.

4 Which of the following is the particular solution of $\frac{dy}{dx} = y^2$ with $y(0) = \frac{1}{2}$, and its domain?

A. $y = \frac{1}{2-x}$, $x < 2$
B. $y = \frac{1}{2-x}$, $x \neq 2$
C. $y = \frac{1}{2+x}$, $x > -2$
D. $y = 2 - x$, all x

7.7 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

Consider $\frac{dy}{dx} = \frac{y}{x^2}$ with $y(1) = e$.

(a) Find the particular solution.

(b) State its domain. Explain your reasoning.

7.7 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

Consider $\frac{dy}{dx} = x\left(1 + y^2\right)$ with $y(0) = 1$.

(a) Find the particular solution.

(b) Find its domain.

7.7 / AP-style free response

Problem 7 / *Show the work and the reasons a grader looks for.*

A roast is taken out of an oven at 350°F and placed in a room at 70°F. Its temperature T, in degrees Fahrenheit, t minutes later satisfies $\frac{dT}{dt} = -\frac{1}{5}(T - 70)$, with $T(0) = 350$. Find $T(t)$.

8 Let $y = f(x)$ be the solution of the differential equation $\frac{dy}{dx} = \cos\left(x^3\right)$ with the initial condition $f(0) = 2$. Which of the following is $f(x)$?

A. $2 + \int_0^x \cos\left(t^3\right) dt$
B. $\int_0^x \cos\left(t^3\right) dt$
C. $2 + \int_2^x \cos\left(t^3\right) dt$
D. $2 + \sin\left(x^3\right)$

7.7 / AP-style free response

Problem 9 ➤ */ Show the work and the reasons a grader looks for.*

A student solves $\dfrac{dy}{dx} = 2xy$ with $y(0) = 5$.
Line 1: $\dfrac{1}{y}\,dy = 2x\,dx$
Line 2: $\ln|y| = x^2 + C$
Line 3: $y = e^{x^2} + e^C$
Line 4: $5 = 1 + e^C$, so $e^C = 4$ and $y = e^{x^2} + 4$

(a) Which line has the first error? Explain your reasoning.

(b) Find the correct particular solution.

7.7 / Answer key

Finding Particular Solutions Using Initial Conditions and Separation of Variables

1 $y = -\sqrt{x^2 + 9}$

2 (A)

3 $y = \sqrt[3]{x^2 + 2x + 5}$

4 (A)

5 (a) $y = e^{2-\frac{1}{x}}$; (b) $x > 0$: $\frac{1}{x}$ breaks at $x = 0$, and the point has $x = 1 > 0$

6 (a) $y = \tan\left(\frac{x^2}{2} + \frac{\pi}{4}\right)$; (b) $-\sqrt{\frac{\pi}{2}} < x < \sqrt{\frac{\pi}{2}}$

7 $T(t) = 70 + 280e^{-t/5}$

8 (A)

9 (a) Line 3: $e^{x^2+C} = e^C \cdot e^{x^2}$, not $e^{x^2} + e^C$; (b) $y = 5e^{x^2}$