Lecture Notes · Unit 7 · Lecture 6 AP CED 7.7

Particular

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INTRO: ONE POINT, ONE CURVE

Particular solutions (AP 7.7)

Let's get that 5

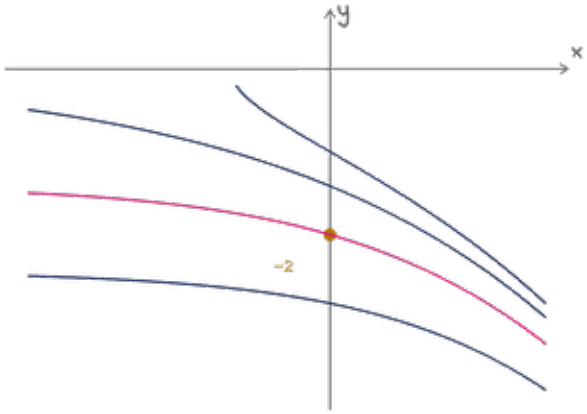
IDEA 1: + C, THEN THE POINT

Key ideas

A family + one point → which curve?

1. + C, then the point

$\frac{dy}{dx} = \frac{e^x}{y}$, $y(0) = -2 \rightarrow y\,dy = e^x\,dx$



$\frac{y^2}{2} = e^x + C \rightarrow +C$ now

$(0,-2) \rightarrow \frac{(-2)^2}{2} = 2,\ e^0 = 1 \rightarrow 2 = 1 + C,\ C = 1$

AP 2023 part → no +C → at most 2 of 4

IDEA 2: THE POINT PICKS THE SIGN

Key ideas

2. The starting point decides

$y^2 = 2e^x + 2 \rightarrow y = \pm\sqrt{2e^x + 2}$

Two branches → The point picks the sign → below

$y = -\sqrt{2e^x + 2}$

ABSOLUTE VALUE: SAME MOVE

Key ideas

Absolute value: same move

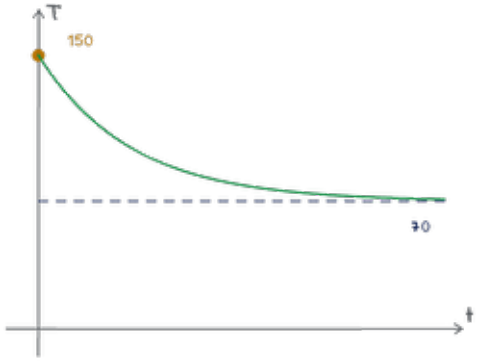
$\frac{dT}{dt} = -\frac{1}{2}(T - 70)$

$T(0) = 150 \rightarrow \frac{dT}{T-70} = -\frac{1}{2}\,dt$

$\ln|T-70| = -\frac{t}{2} + C \rightarrow \ln(150 - 70) = C = \ln 80$

$|T-70| = 80e^{-\frac{t}{2}}$; $150 > 70 \rightarrow +$

$T = 70 + 80e^{-\frac{t}{2}}$ $T(0) = 150 \rightarrow 70$



IDEA 3: WHERE IT LIVES

Key ideas

3. Where it lives

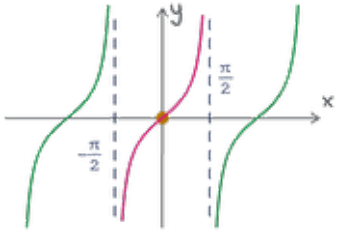
$\frac{dy}{dx} = 1 + y^2$

$\frac{dy}{dx} = 1 + y^2,\ y(0) = 0$

$\frac{dy}{1+y^2} = dx \rightarrow \arctan y = x + C$

$(0,0) \to C = 0 \to y = \tan x$

$\tan x \to$ many pieces



$-\frac{\pi}{2} < x < \frac{\pi}{2} \to$ point's interval $\to$ no jumps

$\to$ particular solution · point = initial condition

IDEA 4: ONLY X? START + INTEGRAL

Key ideas

4. only x

$\frac{dy}{dx} = \sin(x^2)$

$y(1) = 3 \to y = 3 + \int_1^x \sin(t^2)\,dt$

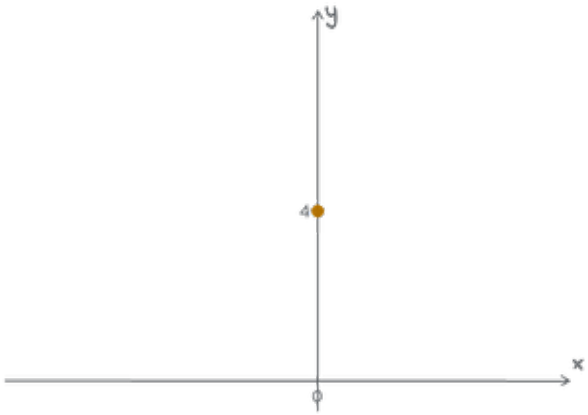
$x = 1 \to \int_1^1 \sin(t^2)\,dt = 0 \to y = 3$ ✓

$\frac{dy}{dx} = \sin(x^2)$ ✓ $\to$ start + change

PROBLEM 1 OF 2

A particular solution

Find the particular solution $y = f(x)$ to $\frac{dy}{dx} = (2x+1)\sqrt{y}$ with the initial condition $f(0) = 4$.



Solution

Separate, integrate with + C at once, the point gives C, then solve for y

Plan: separate $\to$ +C $\to$ point $\to$ y

$\frac{dy}{\sqrt{y}} = (2x+1)\,dx \to y^{-\frac{1}{2}}\,dy = (2x+1)\,dx$

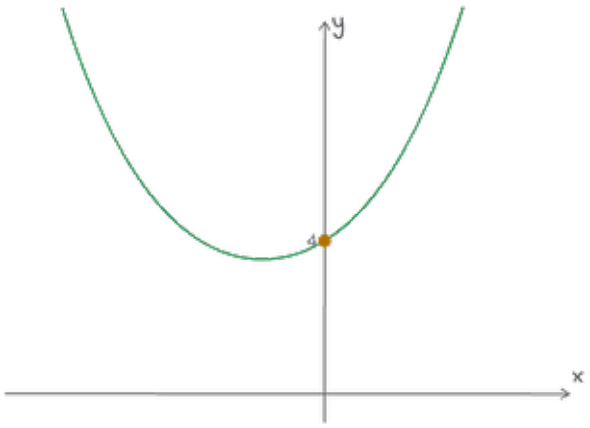
$2\sqrt{y} = x^2 + x + C$

$(0,4) \to 2 \cdot 2 = 0 + C \to C = 4$

Next $\to$ solve for y

Solution

Separate, integrate with + C at once, the point gives C, then solve for y



Plan: y, then check

$$\sqrt{y} = \frac{x^2+x+4}{2}$$

$$f(x) = \left(\frac{x^2+x+4}{2}\right)^2 \rightarrow f(0) = 4 \checkmark$$

Plus C first: workbook

Careful

X Common mistake

$2\sqrt{y} = x^2 + x$ (no C)

Then "+4" at the end

✓ Do this

+C with $\int$

$(0, 4) \rightarrow C = 4$

solve for y

PROBLEM 2 OF 2

Solution and domain

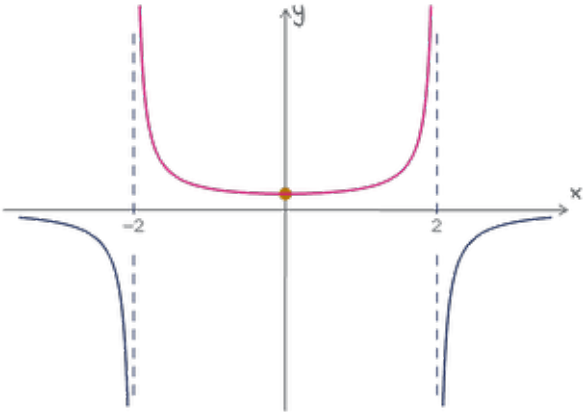
Consider $\frac{dy}{dx} = 2xy^2$ with $y(0) = \frac{1}{4}$. Which is the particular solution, with its domain?

(A) $\frac{1}{4-x^2}$, $x \neq \pm 2$

(B) $\frac{1}{4-x^2}$, $-2 < x < 2$

(C) $\frac{1}{x^2+4}$, all x

(D) $4 - x^2$, all x



Solution

Separate, + C, the point, then the piece through the point

3 moves: +C → point → interval

$$y^{-2}dy = 2x\,dx \rightarrow -\frac{1}{y} = x^2 + C$$

$$\left(0, \frac{1}{4}\right) \rightarrow -4 = 0 + C \rightarrow C = -4$$

$$-\frac{1}{y} = x^2 - 4 \rightarrow y = \frac{1}{4-x^2}$$

$4 - x^2 = 0 \rightarrow x = \pm 2 \rightarrow$ 3 intervals

$-2 < x < 2 \rightarrow$ middle interval

So (B).

Find the piece: workbook

WRAP-UP



Which curve? The point

+C the moment you integrate

The point → C

Point → sign, interval → one curve

Only x → start + $\int$

Exam → +C on the integration line

Workbook → video → comments

Next: 7.8 → exponential models

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.