7.7 / Finding Particular Solutions Using Initial Conditions and Separation of Variables

Watch the lesson ➤

Separate, +C right away, put the point in, solve for y; the point picks the sign and the piece

Learning goal

Find a particular solution from an initial condition, choose the sign and the interval that the point picks, and write a solution as the start plus an integral.

Key ideas

01 A **particular solution** is the one solution through the initial condition.

02 Order: separate → integrate with $+C$ **right away** → put the point in → solve for y.

03 Two branches (± square root, | |): the point picks the sign.

04 The solution lives on one interval: it holds the point and does not jump across a break.

05 When $\frac{dy}{dx}$ is in x only: $y = y_0 + \int_{x_0}^{x} (\text{rate})\, dt$, the start plus the change.

Remember

initial condition
The given point, such as $f(0) = 4$.

particular solution
The one solution that passes through the initial condition.

domain of the solution
The one interval that holds the point.

Class notes

7.7 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Show the point going in: "$(0, 4)$: $2\sqrt{4} = C$, so $C = 4$."	Do not add $+C$ at the end, after solving for y.
State the domain as one interval: $-2 < x < 2$.	Do not write the domain as $x \neq \pm 2$: the solution cannot jump across a break.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 + C at the end

Code DE9

✗ $y^2 = 2e^x$, so $y = \sqrt{2e^x} + C$.

✓ $\frac{y^2}{2} = e^x + C$ first, then the point, then solve for y.

02 The point picks the sign

Code DE11

✗ $y^2 = x^2 + 9$ with $y(0) = -3$: $y = \sqrt{x^2 + 9}$.

✓ $y(0) = -3 < 0$: $y = -\sqrt{x^2 + 9}$.

03 One interval

Code DE11

✗ $y = \tan x$ with $y(0) = 0$: all $x \neq \frac{\pi}{2} + n\pi$.

✓ The interval that holds 0: $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

- 2023 AB FRQ 3(d) – particular solution: separation, the constant, the initial condition

7.7 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Particular solution	Separate; integrate with $+C$ right away; put the point in; solve for y (the point picks the sign).	E01 · Practice 1, 2, 3, 7, 9
Where the solution lives	Find where the formula breaks; the domain is the one interval that holds the point.	E02 · Practice 4, 5, 6
A solution written as an integral	$y = y_0 + \int_{x_0}^{x} \frac{dy}{dx}\,dt$: the start plus the change.	Practice 8

Example 01

Video · E01

Find the particular solution $y = f(x)$ to $\frac{dy}{dx} = (2x + 1)\sqrt{y}$ with the initial condition $f(0) = 4$.

$+C$ right after you integrate, then the point.

Example 02

Video · E02

$\frac{dy}{dx} = 2xy^2$, $y(0) = \frac{1}{4}$. The particular solution and its domain?

A. $\frac{1}{4-x^2}$, $x \neq \pm 2$

B. $\frac{1}{4-x^2}$, $-2 < x < 2$

C. $\frac{1}{x^2+4}$, all x

D. $4 - x^2$, all x

> Find the solution, then the one interval that holds $x = 0$.

VIDEO COMPLETE Check your example answers at the end of this booklet.

7.7 / Example answers

Finding Particular Solutions Using Initial Conditions and Separation of Variables

E01 $f(x) = \left(\frac{x^2+x+4}{2}\right)^2$

E02 (B)