7.6 / Practice workbook

Finding General Solutions Using Separation of Variables

Practice

Problems 1–9 / Show your reasoning

1 Find the general solution of $\frac{dy}{dx} = \frac{4x^3}{y}$. You may leave it in implicit form.

2 Which is the general solution of $\frac{dy}{dx} = y \sec^2 x$?

A. $y = Ae^{\tan x}$
B. $y = \tan x + C$
C. $y = e^{\tan x} + C$
D. $y = A \tan x$

3 Find the general solution of $\frac{dy}{dx} = \frac{y+1}{x}$ for $x > 0$. Write y as a function of x.

4 Which is the general solution of $\frac{dy}{dx} = y^2 \sin x$?

A. $y = \frac{1}{\cos x + C}$
B. $y = -\frac{1}{\cos x} + C$
C. $y = \frac{1}{C - \cos x}$
D. $y = \frac{y^3}{3} \sin x + C$

5 Which is the general solution of $\frac{dy}{dx} = 2xe^y$?

A. $y = -\ln(C - x^2)$
B. $y = \ln(x^2 + C)$
C. $y = -\ln(x^2) + C$
D. $y = x^2 e^y + C$

7.6 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

Consider $\frac{dy}{dx} = \frac{x+1}{xy^2}$ for $x > 0$.

(a) Separate the variables and find the general solution in implicit form.

(b) Write y as a function of x.

7.6 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

Consider the differential equation $\frac{dy}{dx} = 2xy + 4x$.

(a) Find the general solution. Write y as a function of x.

(b) To separate, you divided by $y + 2$. Is the constant function $y = -2$ a solution? If so, which value of the constant in (a) gives it?

8 Which is the general solution of the differential equation $\frac{dy}{dx} = e^{2x+y}$?

A. $y = -\ln\left(C - \frac{1}{2}e^{2x}\right)$

B. $y = \ln\left(\frac{1}{2}e^{2x} + C\right)$

C. $y = -\ln\left(C - e^{2x}\right)$

D. $y = \frac{1}{2}e^{2x+y} + C$

9 Which of the following differential equations can NOT be solved by separation of variables?

A. $\frac{dy}{dx} = e^{x+y}$

B. $\frac{dy}{dx} = x + y^2$

C. $\frac{dy}{dx} = \frac{y}{x}$

D. $\frac{dy}{dx} = xy + x$

7.6 / Answer key

Finding General Solutions Using Separation of Variables

1 $\frac{y^2}{2} = x^4 + C$

2 (A)

3 $y = Ax - 1$

4 (A)

5 (A)

6 (a) $\frac{y^3}{3} = x + \ln x + C$; (b) $y = \sqrt[3]{3x + 3\ln x + C}$

7 (a) $y = Ae^{x^2} - 2$; (b) yes: $\frac{dy}{dx} = 0$ and $2x(-2) + 4x = 0$; it is $A = 0$

8 (A)

9 (B)