Lecture Notes · Unit 7 · Lecture 5 AP CED 7.6

Separation

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: SEPARATE FIRST

Separation of variables (AP 7.6)

Let's get that 5

IDEA 1: Y WITH DY, X WITH DX

Key ideas

$\frac{dy}{dx} = \frac{x^2}{y^2}$ → integrate? y is on the right

1. y with dy, x with dx

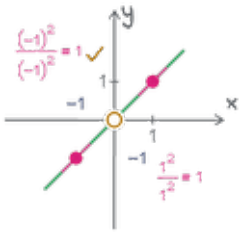
Multiply by $y^2\,dx$ → $y^2\,dy = x^2\,dx$

$\int y^2\,dy = \int x^2\,dx$ → $\frac{y^3}{3} = \frac{x^3}{3} + C$

Not a fraction → why split?

$\frac{d}{dx}\frac{y^3}{3} = y^2\frac{dy}{dx} = x^2$ ✓

$C = 0$ → $y^3 = x^3$ → $y = x$, $y \neq 0$



→ separation of variables

AP 2023 part → no separation → 0 of 4

IDEA 2: ONE + C, A FAMILY

Key ideas

2. one + C

$\frac{dy}{dx} = \frac{x^2}{y^2}$

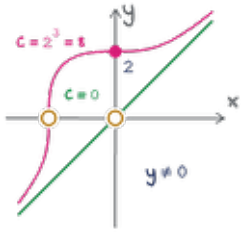
$\frac{y^3}{3} + C_1 = \frac{x^3}{3} + C_2$ → two constants

$C_2 - C_1$ → just one C, on one side

$y^3 = x^3 + 3C$, $3C$ → just C

$y = \sqrt[3]{x^3 + C}$ → a general solution

$C = 0$ → $y = x$, $C = 8$ → $y = \sqrt[3]{x^3 + 8}$, (0, 2)



the whole family

IDEA 3: 1/Y → LN

Key ideas

3. one over y → natural log

$\frac{dy}{dx} = y\cos x$

$y \neq 0$ → $\frac{1}{y}\,dy = \cos x\,dx$

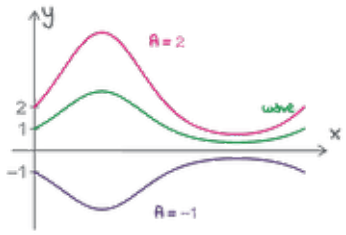
$\ln|y| = \sin x + C$

$|y| = e^C e^{\sin x}$ → $\pm e^C = A$ → $y = Ae^{\sin x}$

$y = 0$ → $\frac{dy}{dx} = 0$ $y\cos x = 0 \cdot \cos x = 0$ → a solution

$A = 0$ → $y = 0 \cdot e^{\sin x} = 0$ → already in ✓

$A = 1$



Same steps: separate, integrate, one C

IDEA 4: A PRODUCT, NOT A SUM

Key ideas

4. A product? Then separate

$\frac{dy}{dx} = xy + 3x = x(y+3) \rightarrow \frac{dy}{y+3} = x\,dx \quad y \neq -3$

$\frac{dy}{dx} = x + y \rightarrow$ a sum → cannot separate

PROBLEM 1 OF 2

A general solution

Find the general solution of $\frac{dy}{dx} = \frac{2x}{\cos y}$.

You may leave it in implicit form.

Solution

Separate (y with dy, x with dx), integrate both sides, one + C

Separate → $\int$ → +C

$\cos y\,dy = 2x\,dx$

$\int \cos y\,dy = \int 2x\,dx$

$\sin y = x^2 + C \quad$ (1 C)

So $\sin y = x^2 + C$ is the general solution.

Separate first: workbook

Careful → separate first

X Common mistake

$\int \frac{2x}{\cos y} dx$

$y = \frac{x^2}{\cos y} + C$

✓ Do this

$\cos y\,dy = 2x\,dx$

$\int \cos y\,dy = \int 2x\,dx$

PROBLEM 2 OF 2

Which general solution?

Which is the general solution of $\frac{dy}{dx} = e^{x-y}$?

(A) $y = x + C$

(B) $y = \ln(e^x + C)$

(C) $y = -\ln(C - e^x)$

(D) $y = e^x + C$

Solution

Split the exponent, separate, integrate, then solve for y

e^{x-y} → quotient → separate, integrate → y

$e^{x-y} = \frac{e^x}{e^y} \rightarrow e^y\,dy = e^x\,dx$

$e^y = e^x + C$ now

$y = \ln(e^x + C)$

So (B).

Split, then separate: workbook

WRAP-UP



How? Separate first

y with dy, x with dx

Integrate each side → one +C

The C → a whole family

A product, not a sum → separate

Exam → show the separation line

Workbook → video → comments

Next: 7.7 → one point, one curve

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.