7.6 / Finding General Solutions Using Separation of Variables Watch the lesson ➤

y with dy, x with dx; integrate each side; one +C; a new name for the constant

Learning goal

Find the general solution of a separable differential equation: separate, integrate each side, add one constant, and solve for y when you can.

Key ideas

01 **Separation of variables**: multiply or divide so every y is with dy and every x is with dx. Then integrate each side.

02 One $+C$: two constants combine into one, written the moment you integrate. With C left in, it is the **general solution**.

03 y in the bottom gives $\ln|y|$. Raise e to both sides: $e^{\sin x+C} = e^C e^{\sin x}$, and $\pm e^C$ gets a new name, A.

04 Dividing by y (or $y+3$) assumes it is not 0. Check that constant function: it is often $A = 0$.

05 A sum such as $x+y$ cannot be separated. Factor or split first: $xy+3x = x(y+3)$, $e^{x-y} = e^x e^{-y}$.

Remember

separable
The right side is (a function of x) times (a function of y).

general solution
All the solutions, with the constant left in.

implicit form
An equation in x and y, not solved for y.

Class notes

7.6 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Show the separated line, $\cos y\,dy = 2x\,dx$, before integrating.	Do not integrate $\frac{dy}{dx} = x^2y$ as $y = \frac{x^3}{3}y + C$: separate first.
Write $+C$ on the line where you integrate.	Do not write $e^{x^2+C} = e^{x^2} + e^C$.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Separate before you integrate

Code DE8

✗ $\frac{dy}{dx} = xy$, so $y = \frac{x^2}{2}y + C$.

✓ $\frac{1}{y}\,dy = x\,dx$ first; then $\ln|y| = \frac{x^2}{2} + C$.

02 e to a sum is a product

Code DE10

✗ $\ln|y| = x^2 + C$, so $y = e^{x^2} + C$.

✓ $|y| = e^{x^2+C} = e^C e^{x^2}$, so $y = Ae^{x^2}$.

03 The + C goes in at once

Code DE9

✗ $\frac{y^3}{3} = \frac{x^3}{3}$, $y = x$, then $y = x + C$.

✓ $\frac{y^3}{3} = \frac{x^3}{3} + C$, so $y = \sqrt[3]{x^3 + C}$.

Real AP questions on this topic Free on AP Central. Try them after the practice.

- 2023 AB FRQ 3(d) – separation of variables with an initial condition

7.6 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Separate, integrate, one $+C$	y with dy, x with dx; integrate each side; one $+C$ right away; solve for y if asked.	E01, E02 · Practice 1, 2, 3, 4, 5, 6, 8, 9
Factor first; the constant solution you divided away	Factor the right side into (x-part)·(y-part); check the constant you divided by.	Practice 7

Example 01

Video · E01

Find the general solution of $\frac{dy}{dx} = \frac{2x}{\cos y}$.
You may leave it in implicit form.

> Multiply by $\cos y\,dx$ so every y goes with dy.

Example 02

Video · E02

Which is the general solution of $\frac{dy}{dx} = e^{x-y}$?

> Split the exponent: $e^{x-y} = e^x\,e^{-y}$.

A. $y = x + C$
B. $y = \ln(e^x + C)$
C. $y = -\ln(C - e^x)$
D. $y = e^x + C$

VIDEO COMPLETE Check your example answers at the end of this booklet.

7.6 / Example answers

Finding General Solutions Using Separation of Variables

E01 $\sin y = x^2 + C$

E02 (B)