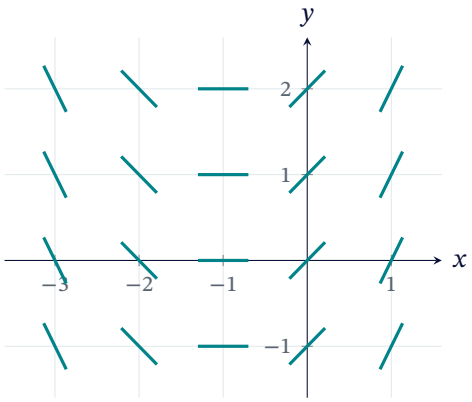
7.4 / Practice workbook

Reasoning Using Slope Fields and Differential Equations

Practice

Problems 1–8 / Show your reasoning

1 The slope field of $\frac{dy}{dx} = x + 1$ is shown. Which could be the solution through $(-1, 0)$?

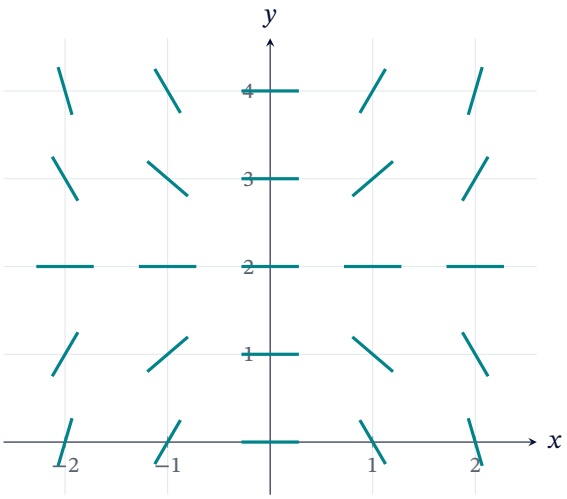


A. $y = \frac{1}{2}(x+1)^2$
B. $y = x + 1$
C. $y = -\frac{1}{2}(x+1)^2$
D. $y = e^{x+1} - 1$

7.4 / AP-style free response

Problem 2 ➤ / *Show the work and the reasons a grader looks for.*

The slope field for the differential equation $\frac{dy}{dx} = x(y-2)$ is shown at the points with $x = -2, -1, 0, 1, 2$ and $y = 0, 1, 2, 3, 4$.



(a) Sketch the solution curve that passes through the point $(0, 3)$.

(b) Does this solution have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

3 Let $y = f(x)$ be the solution to $\frac{dy}{dx} = xy - 1$ with $f(2) = 1$. Write an equation for the line tangent to the graph of f at $x = 2$, and use it to approximate $f(2.1)$.

4 Consider $\frac{dy}{dx} = x^2(y - 2)$. Which statement is true?

A. $y = 2$ is a constant solution.

B. Every solution is increasing.

C. The solution through $(0, 3)$ has a relative minimum at $x = 0$.

D. The solution through $(1, 1)$ is increasing at $x = 1$.

7.4 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

Let $y = f(x)$ be the solution to $\frac{dy}{dx} = 3 - xy$ with $f(0) = 2$.

(a) Use the tangent line at $x = 0$ to approximate $f(0.1)$.

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Is the graph of f concave up or down at $(0, 2)$?

6 Let $y = f(x)$ be the solution to $\frac{dy}{dx} = (x-1)e^y$ whose graph passes through $(1, 0)$. At $x = 1$, f has

A. a relative minimum
B. a relative maximum
C. neither a relative maximum nor a relative minimum
D. not enough information to decide

7.4 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

Let $y = f(x)$ be the solution to $\frac{dy}{dx} = y^2(3 - x)$ with $f(0) = 1$.

(a) Use the tangent line at $x = 0$ to approximate $f(0.2)$.

(b) Find $\frac{d^2y}{dx^2}$. Is the approximation an overestimate or an underestimate? Explain your reasoning.

(c) Another solution passes through $(3, 2)$. Does it have a relative maximum, a relative minimum, or neither there? Justify your answer.

8 Consider $\frac{dy}{dx} = y(y-2)^2$. Which of the following are true?
I. $y = 2$ is a constant solution.
II. The solution with $y(0) = 1$ is increasing at $x = 0$.
III. The solution with $y(0) = 1$ is concave up at $x = 0$.

A. I and II only
B. I and III only
C. II and III only
D. I, II, and III

7.4 / Answer key

Reasoning Using Slope Fields and Differential Equations

1 (A)

2 (a) a valley with its lowest point at $(0, 3)$, above the line $y = 2$; (b) relative minimum: $\frac{dy}{dx} = 0$ at $(0, 3)$ and changes from $-$ to $+$ (the curve stays above $y = 2$)

3 $y = 1 + (x - 2)$; $f(2.1) \approx 1.1$

4 (A)

5 (a) $y = 2 + 3x$; $f(0.1) \approx 2.3$; (b) $\frac{d^2y}{dx^2} = -y - x(3 - xy)$; $-2 < 0$ at $(0, 2)$: concave down

6 (A)

7 (a) $y = 1 + 3x$; $f(0.2) \approx 1.6$; (b) underestimate: $\frac{d^2y}{dx^2} = y^2\left(2y(3 - x)^2 - 1\right) > 0$ on $0 \leq x \leq 0.2$; (c) relative maximum: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = -4 < 0$ at $(3, 2)$

8 (A)