7.2 / Practice workbook

Verifying Solutions for Differential Equations

Practice

Problems 1–7 / Show your reasoning

1 Which function is a solution of the differential equation $\frac{dy}{dx} = -2xy$?

A. $y = e^{-x^2}$

B. $y = e^{-2x}$

C. $y = -x^2$

D. $y = e^{x^2}$

2 Show that $y = 3e^{2x} + 1$ is a solution of the differential equation $\frac{dy}{dx} = 2y - 2$ with the initial condition $y(0) = 4$.

3 For what value of k is $y = kx^2 + x$ a solution of $x\frac{dy}{dx} - y = 4x^2$?

A. 1

B. 2

C. 4

D. 8

7.2 / AP-style free response

Problem 4 ➤ */ Show the work and the reasons a grader looks for.*

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) Show that $y = Ce^{-x} + 2x - 2$ is a solution for every constant C.

(b) Find the solution whose graph passes through $(0, 1)$.

5 Which of the following are solutions of $y'' = 4y$?

I. $y = e^{2x}$

II. $y = e^{-2x} + 3$

III. $y = 5e^{-2x}$

A. I only

B. I and II only

C. I and III only

D. I, II, and III

7.2 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

Consider the differential equation $\frac{dy}{dx} = -y^2$.

(a) Show that $y = \frac{1}{x + C}$ is a solution for every constant C.

(b) Find the solution with $y(1) = \frac{1}{3}$.

(c) Is $y = \frac{2}{x + C}$ also a solution? Explain your reasoning.

7 Find all values of k for which $y = e^{kx}$ is a solution of the differential equation $y'' - y' - 6y = 0$.

7.2 / Answer key

Verifying Solutions for Differential Equations

1 (A)

2 both sides are $6e^{2x}$ for every x, and $y(0) = 4$

3 (C)

4 (a) both sides are $-Ce^{-x} + 2$; (b) $y = 3e^{-x} + 2x - 2$

5 (C)

6 (a) both sides are $-\frac{1}{(x+C)^2}$; (b) $y = \frac{1}{x+2}$; (c) no: $y' = -\frac{2}{(x+C)^2}$, but $-y^2 = -\frac{4}{(x+C)^2}$

7 $k = 3$ and $k = -2$