Lecture Notes · Unit 7 · Lecture 2 AP CED 7.2

Verifying solutions of differential equations

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INTRO: SOMEONE HANDS YOU A FUNCTION

Checking a Solution AP Calculus 7.2

Function: "it's a solution!"

$\frac{dy}{dx} = x + y \qquad y'' + 9y = 0$

No method → not needed

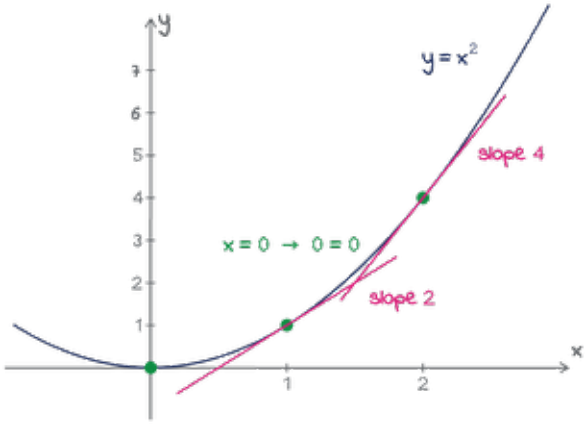
Solution? Solve it first?

Check every x, not 1 point

A FUNCTION THAT WORKS

Is $y = x^2$ a solution?

$x\frac{dy}{dx} = 2y \rightarrow$ asks $\frac{dy}{dx} = \frac{2y}{x}, x \neq 0$



$(1, 1) \rightarrow$ asks $\frac{2 \cdot 1}{1} = 2 \rightarrow$ slope 2

$(2, 4) \rightarrow \frac{2 \cdot 4}{2} = 4 \rightarrow$ slope 4

Two matches → proof? No: stopped clock

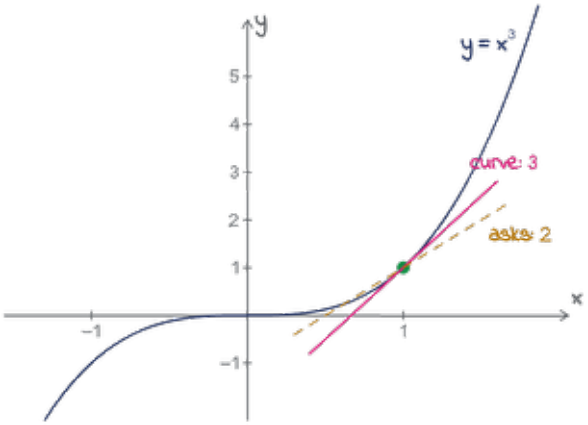
Every $x \rightarrow y' = 2x$

$x \cdot 2x = 2x^2 = 2y$ ✓ → even at $x = 0$

True for every $x \rightarrow y = x^2$ is a solution

A FUNCTION THAT FAILS

Is $y = x^3$ a solution?



Now $y = x^3$ in $x\frac{dy}{dx} = 2y$

$(1, 1) \rightarrow$ curve 3, asks $\frac{2 \cdot 1}{1} = 2$ X

Every $x \rightarrow y' = 3x^2$

$3x^2 = \frac{2y}{x} (x \neq 0) \rightarrow y = \frac{3}{2}x^3$

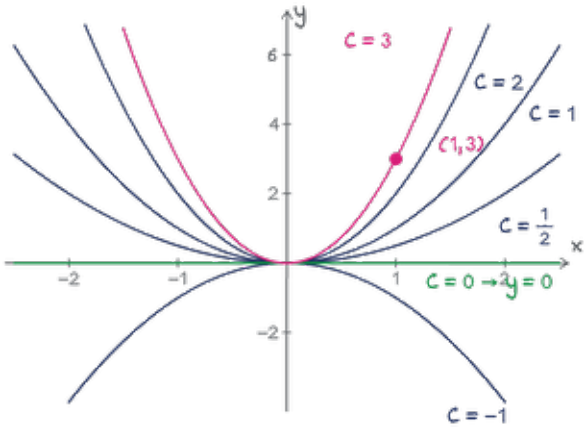
$\frac{3}{2}x^3 \neq x^3 \rightarrow$ not a solution

A WHOLE FAMILY OF SOLUTIONS

Solutions of $x\frac{dy}{dx} = 2y$

$y = Cx^2 \rightarrow y' = 2Cx$

$x \cdot 2Cx = 2Cx^2 = 2y$ ✓ for every C



$C = 0 \rightarrow y = 0 \rightarrow x \cdot 0 = 2 \cdot 0$ ✓ constant

Every C ✓ → a family of solutions

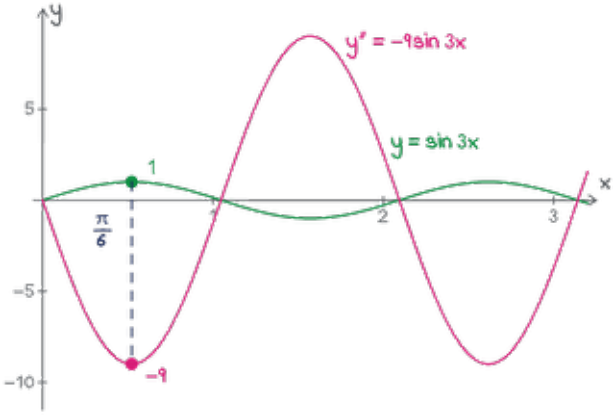
$(1,3) \rightarrow C \cdot 1^2 = 3 \rightarrow C = 3$

$(1,3)$ → one curve from this family

WHEN Y'' SHOWS UP

$y'' + 9y = 0$

$y'' + 9y = 0$ → asks $y'' = -9y$



Try $y = \sin 3x$ → finding it not on AP

$y' = 3\cos 3x$

$y'' = -9\sin 3x = -9y$

$-9\sin 3x + 9\sin 3x = 0$ every x

$y = \sin 3x$ → solution

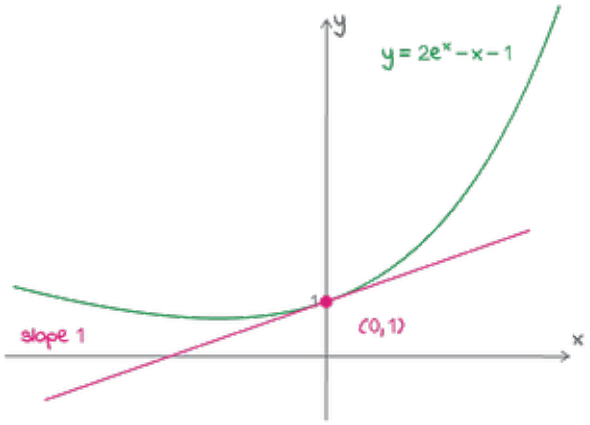
y'' → two derivatives, same check

AP "verify ..." → plug in, sides match + initial condition

PROBLEM 1 OF 2

Verify it, then the point

Show that $y = 2e^x - x - 1$ is a solution of the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 1$.



Solution Both sides, then the point

Plan: left, right → same? → point

Left: $y' = 2e^x - 1$

Right: $x + (2e^x - x - 1) = x + 2e^x - x - 1 = 2e^x - 1$

Same expression → every x ✓

$y(0) = 2e^0 - 0 - 1 = 1$ ✓ slope $0 + 1 = 1$

So $y = 2e^x - x - 1$ is a solution, $y(0) = 1$

Workbook: both sides, then the point

THE ONE-POINT TRAP

One point can fool you

Fast check: $y(0) = 1$ ✓ → done?

Try $y = x + 1$

$y(0) = 1$ ✓

$y'(0) = 1$, $x + y = 1$ ✓

Every x → left 1, right $x + (x + 1) = 2x + 1$

$1 = 2x + 1 \rightarrow x = 0$ only → not a solution

A stopped clock → one point only

Same expression → every x; then $y(0)$: two checks

PROBLEM 2 OF 2

Which r works?

For which values of r is $y = x^r$ a solution of $x^2y'' - 2y = 0$ for $x > 0$?

(A) 2 only

(B) −1 only

(C) 2 and −1

(D) −2 and 1

Solution Two derivatives, plug in, solve for r

y', y'' → plug in → solve r ($x > 0$)

$y' = rx^{r-1} \rightarrow y'' = r(r-1)x^{r-2}$

$x^2y'' = x^2 \cdot r(r-1)x^{r-2} = r(r-1)x^r$

$x^2y'' - 2y = r(r-1)x^r - 2x^r = (r(r-1) - 2)x^r = 0 \rightarrow x^r \neq 0 \rightarrow r^2 - r - 2 = 0$

$(r-2)(r+1) = 0 \rightarrow r = 2, -1$ both

So (C)

Workbook: y'' → plug in → solve

WRAP-UP

Is it a solution?

Never solved the equation: plug it in → true for every x on the interval → a solution

One point → a stopped clock

Can be a family; (1, 3) picked $C = 3$ from it

y'' → two derivatives

Next → 7.3: slope fields

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.