7.2 / Verifying Solutions for Differential Equations

Watch the lesson ➤

Put it in: both sides match for every x; a family, and a point picks one

Learning goal

Decide whether a function is a solution of a differential equation by putting it in, without solving, and use a point to pick one solution from a family.

Key ideas

01 A **solution** makes the equation true for **every** x in an interval, not at one point.

02 To verify: find y' (and y''), put them in, simplify each side by itself, and end with a sentence: "so it is a solution."

03 With y'' in the equation: take two derivatives and put both in. You check it; you do not solve it.

04 A constant C can give a whole **family** of solutions. A point puts in x and y and picks one value of C.

05 To make a function a solution, put it in and factor out the part that is never 0 (such as x^r for $x > 0$), then solve for the constant.

Remember

solution
A function that makes the differential equation true for every x in an interval.

initial condition
A given point such as $y(0) = 1$; it picks one solution.

verify
Put the function in and show both sides are the same expression.

Class notes

7.2 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Show both sides: "$y' = 2e^x - 1$ and $x + y = 2e^x - 1$." Then write the conclusion.	Do not test one value of x and call it a solution.
Say why you may divide: "$x^r \neq 0$ for $x > 0$."	Do not solve the equation when it asks you to verify: put the function in.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 One point is not enough

Code DE3

✗ $y = x^2$ for $y' = 2y - x^2$? At $x = 0$: $0 = 0$, so it is a solution.

✓ $y' = 2x$ but $2y - x^2 = x^2$: equal only at $x = 0$ and $x = 2$. Not a solution.

02 A point alone does not make a solution

Code DE3

✗ $y = 3x + 1$ passes through $(0, 1)$, so it solves $y' = x + y$, $y(0) = 1$.

✓ Check the equation first: $y' = 3$ but $x + y = 4x + 1$. Not a solution, even through the point.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released free-response part on 7.2 alone; verifying is the check at the end of a separation problem (2023 AB Q3 (d)).

7.2 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Is it a solution?	Find the derivatives, put them in, simplify each side by itself; equal for every x o a solution.	E01 · Practice 1, 2, 5
A family, and a point picks one	Check the family with the constant left as a letter; then put the point in to find it.	Practice 4, 6
Find the constant that makes it a solution	Put the function in, factor out the part that is never 0, solve for the constant.	E02 · Practice 3, 7

Example 01

Video · E01

Show that $y = 2e^x - x - 1$ is a solution of the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 1$.

> Work out each side by itself, then the point.

Example 02

Video · E02

For which values of r is $y = x^r$ a solution of $x^2y'' - 2y = 0$ for $x > 0$?

A. 2 only
B. -1 only
C. 2 and -1
D. -2 and 1

> Two derivatives, put them in, factor out x^r.

VIDEO COMPLETE Check your example answers at the end of this booklet.

7.2 / Example answers

Verifying Solutions for Differential Equations

E01 both sides are $2e^x - 1$ for every x; $y(0) = 1$

E02 (C)