7.1 / Practice workbook

Modeling Situations with Differential Equations

Practice

Problems 1–6 / Show your reasoning

1 In a town of 1000 people, a rumor spreads. The rate at which N, the number of people who have heard it, increases is proportional to the number of people who have not heard it. If k is a positive constant, which differential equation models N?

A. $\frac{dN}{dt} = kN$

B. $\frac{dN}{dt} = k(1000 - N)$

C. $\frac{dN}{dt} = \frac{k}{1000-N}$

D. $\frac{dN}{dt} = k(N - 1000)$

2 The amount Q of a chemical is decreasing at a rate inversely proportional to the square of Q. If k is a positive constant, which differential equation describes Q?

A. $\frac{dQ}{dt} = \frac{k}{Q^2}$

B. $\frac{dQ}{dt} = -\frac{k}{Q^2}$

C. $\frac{dQ}{dt} = -kQ^2$

D. $\frac{dQ}{dt} = -\frac{k}{t^2}$

3 The rate of change of y with respect to x is inversely proportional to x^2. A solution curve passes through $(2, 5)$, where its slope is 3. What is $\frac{dy}{dx}$ at the point $(4, 1)$?

A. $\frac{3}{4}$

B. $\frac{3}{2}$

C. 3

D. 12

4 A baked potato is placed in a room at 70 degrees Fahrenheit. Ana models its temperature T with $\frac{dT}{dt} = a(T - 70)$, and Ben with $\frac{dT}{dt} = b(70 - T)$. When $T = 180$, the potato is cooling at 11 degrees per minute. Which is true?

A. $a = \frac{1}{10}$, $b = -\frac{1}{10}$

B. $a = -\frac{1}{10}$, $b = \frac{1}{10}$

C. $a = b = \frac{1}{10}$

D. $a = b = -\frac{1}{10}$

7.1 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

Water drains from a tank. The table gives the rate R, in cm per minute, at which the depth h of the water, in cm, is decreasing.

h (cm)	1	4	9
R (cm/min)	3	6	9

(a) Is R proportional to h, or to $\sqrt{h}$? Explain your reasoning.

(b) Write a differential equation for $\frac{dh}{dt}$.

(c) Find $\frac{dh}{dt}$ when $h = 25$. Explain the meaning of your answer in the context of the problem.

7.1 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

A bacteria culture grows at a rate proportional to B, the number of bacteria. At the same time, a lab removes bacteria at a constant rate of 50 per hour.

(a) Write a differential equation for $\frac{dB}{dt}$.

(b) When $B = 400$, B is increasing at 30 per hour. Find the constant of proportionality.

(c) Find $\frac{dB}{dt}$ when $B = 200$. Explain the meaning of your answer in the context of the problem.

7.1 / Answer key

Modeling Situations with Differential Equations

1 (B)

2 (B)

3 (A)

4 (B)

5 (a) to $\sqrt{h}$: $\frac{R}{\sqrt{h}} = 3$ each time; (b) $\frac{dh}{dt} = -3\sqrt{h}$; (c) -15: when $h = 25$ cm, the depth is decreasing at 15 cm per minute

6 (a) $\frac{dB}{dt} = kB - 50$; (b) $k = \frac{1}{5}$; (c) -10: when $B = 200$, the number of bacteria is decreasing at 10 per hour