Lecture Notes · Unit 7 · Lecture 1 AP CED 7.1

Modeling with differential equations

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INTRO: A COLD SODA

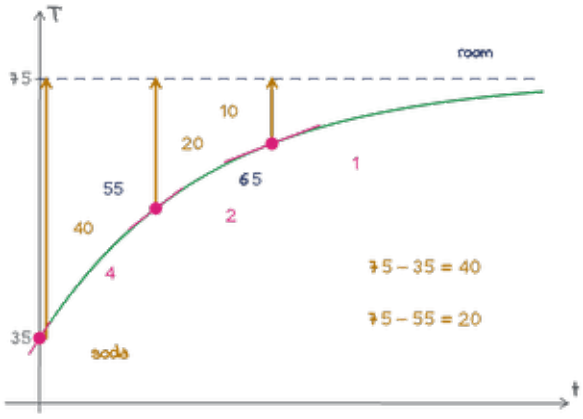
Modeling with Differential Equations AP Calculus 7.1

Cold soda → how fast?

Solve? No. words → one equation

Back to the soda at the end

A RULE FOR THE SLOPE



Room 75°F, soda 35°F

gap = 75 – T

gap 40 → 4°F/min, gap 20 → 2°F/min

Half the gap → half as fast

Rate = 0.1 × gap

$\frac{dT}{dt} = 0.1(75 - T)$

T = 65 → gap 10 → 0.1 · 10 = 1°F/min

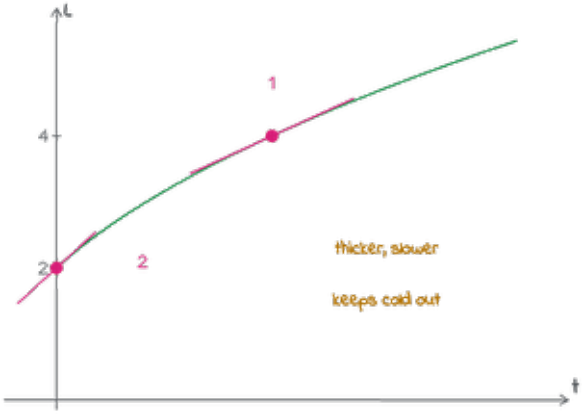
t ?

No clock, no formula → only T → slope

→ a differential equation

WORDS TO AN EQUATION

Words → equation



Rate proportional to gap → k(75 – T), k = 0.1

Ice: L = 2 → 2 cm/day, L = 4 → 1 cm/day

$2 \cdot 2 = 1 \cdot 4 = 4 \rightarrow \frac{dL}{dt} \cdot L = 4$

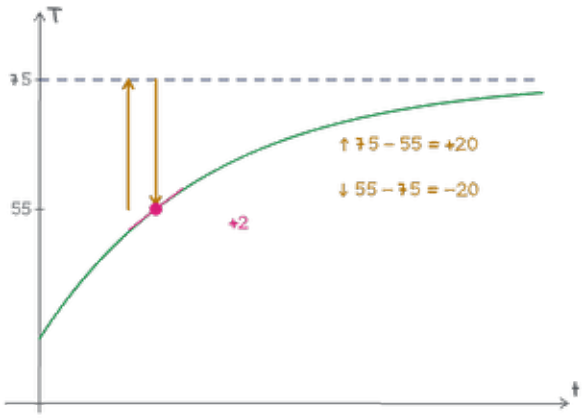
$\frac{dL}{dt} = \frac{4}{L}$ → inversely proportional

Proportional to Q → kQ

Inversely proportional to Q → $\frac{k}{Q}$

y with respect to x → $\frac{dy}{dx}$

THE SIGN



$k(T-75)$, $k = 0.1$? $T = 55 \to -20$, slope $+2$

$k(-20) = 2 \quad k = -0.1 \to$ flipped

AP: decreasing proportional to y, $k > 0$

$\to \frac{dy}{dt} = -ky \ (y > 0)$, the minus in front

AP → proportional to → equation

PROBLEM 1 OF 2

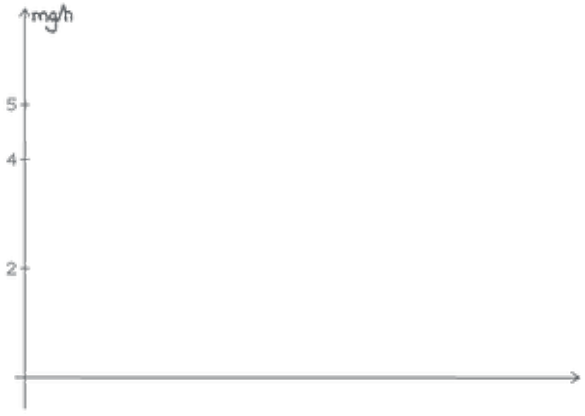
In minus out

A drug enters a patient's body through an IV at a constant rate of 4 mg per hour. The body removes the drug at a rate proportional to A, the amount of the drug in the body, in mg.

(a) Write a differential equation for $\frac{dA}{dt}$.

(b) When $A = 20$, A is increasing at 2 mg per hour. Find the constant of proportionality.

(c) Find $\frac{dA}{dt}$ when $A = 50$. Explain its meaning in context.



Solution (a) In minus out

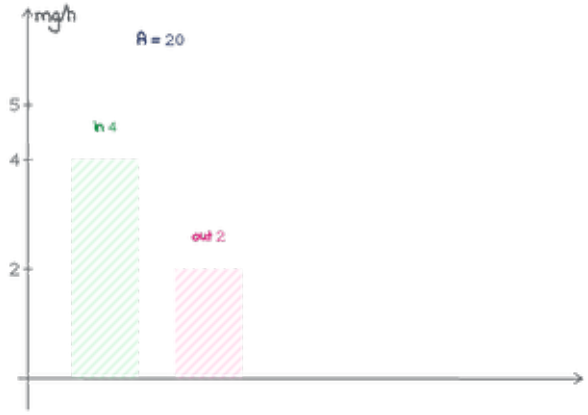
(a)

Plan: in − out → k → plug in A

In 4, out kA → rates

$$\frac{dA}{dt} = 4 - kA, \ k > 0$$

Solution (b) Find the constant



(b)

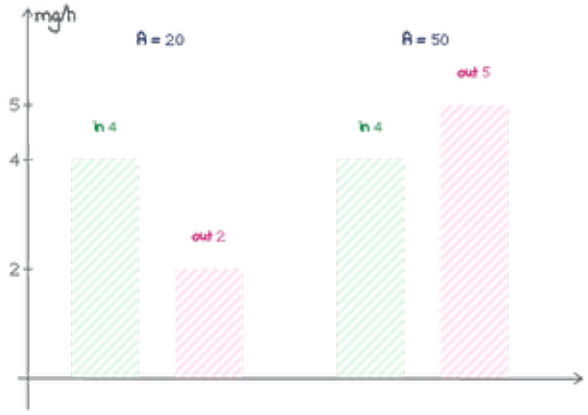
$A = 20 \to$ up $2 = \frac{dA}{dt}$, in 4 → out 2

$$\text{out} = 20k = 2 \ \to \ k = \frac{1}{10}$$

$$\frac{dA}{dt} = 4 - \frac{A}{10}$$

Part (c): $A = 50 \to$ try it, then come back.

Solution (c) Plug in A = 50



(c)

Plan: $A = 50$ → meaning

$\frac{50}{10} = 5$ out, in 4

$4 - 5 = -1$

when $A = 50$ mg → decreasing at 1 mg/h

when, which way, how fast (at 1, not −1)

In minus out: another in your workbook

which variable goes in?

✗ Common mistake

→ $-k\sqrt{t}$, or $-\sqrt{v}$

✓ Do this

$\frac{dv}{dt} = -k\sqrt{v}, k > 0$

that quantity × k

PROBLEM 2 OF 2

k from one point

The rate of change of y with respect to x is proportional to the product of x and y. A solution curve passes through $(1, 2)$, where its slope is 6. For the same equation, what is $\frac{dy}{dx}$ at the point $(2, 3)$?

(A) 6
(B) 9
(C) 12
(D) 18

Solution First point → k → new point

Plan: words → equation → k → (2, 3)

$\frac{dy}{dx} = kxy$

$(1, 2) \rightarrow 6 = k \cdot 1 \cdot 2 \rightarrow k = 3$

$(2, 3) \rightarrow 3 \cdot 2 \cdot 3 = 18$ → not 6 (A): slope from the new point

So (D) → 18.

Workbook → k, then the next point

WRAP-UP

How fast, without a formula?

No formula → a rule for the slope

$\frac{dT}{dt} = 0.1(75 - T)$ → how fast, at any T

words → kQ or $\frac{k}{Q}$

→ then the sign

Next → 7.2: checking a solution

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.