7.1 / Modeling Situations with Differential Equations

Watch the lesson ➤

Words to a differential equation; read the rate without solving

Learning goal

Turn a sentence about a rate into a differential equation, find the constant from one known rate, and read the rate at any amount without solving.

Key ideas

01 A **differential equation** is an equation with a function and its derivative, such as $\frac{dA}{dt} = 4 - kA$.

02 "The rate of change of A with respect to t" is $\frac{dA}{dt}$: the quantity on top, the variable on the bottom.

03 **Proportional** to Q means kQ; **inversely proportional** to Q means $\frac{k}{Q}$. Whatever comes after "proportional to" goes in, whole.

04 In minus out: rate = (rate in) − (rate out). With $k > 0$, "decreasing" puts a minus in front; flipping a gap such as $70 - T$ to $T - 70$ flips the sign of k.

05 No solving needed: put one known amount and its rate in to find k, then put the new amount in. The rate is read from the **amount** right then.

Remember

differential equation
An equation that relates a function and its derivative.

proportional to
Equal to a constant k times that quantity.

constant of proportionality
The number k; one known rate finds it.

Class notes

7.1 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Writing checklist

Notation to use and to avoid

WRITE	AVOID
Write the rate with its letters first: $\frac{dA}{dt} = 4 - kA$. Then put numbers in.	Do not write $\frac{dt}{dA}$ for "the rate of change of A."
Give a rate with units and a sign word: "decreasing at 1 mg per hour."	Do not write "decreasing at -1": the word decreasing already carries the minus.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Inversely: the amount goes in the bottom

Code DE1

✗ Decreasing at a rate inversely proportional to $\sqrt{V}$: $\frac{dV}{dt} = -k\sqrt{V}$.

✓ Inversely puts it in the bottom: $\frac{dV}{dt} = -\frac{k}{\sqrt{V}}$, $k > 0$.

02 Which way the gap points

Code DE2

✗ A pie cools toward 70°: $\frac{dT}{dt} = k(T - 70)$ with $k > 0$.

✓ Cooling while $T > 70$ needs a negative rate: $\frac{dT}{dt} = -k(T - 70)$, or $k(70 - T)$, $k > 0$.

03 The sign word carries the sign

Code DE13

✗ $\frac{dA}{dt} = -1$: "the amount is decreasing at -1 mg per hour."

✓ "The amount is decreasing at 1 mg per hour." Decreasing already says minus.

Real AP questions on this topic Free on AP Central. Try them after the practice.

No released free-response part asks for 7.1 alone; it is the first step of every differential-equation question (for example 2023 AB Q3).

7.1 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Words to a differential equation	The rate is $\frac{d(\text{amount})}{dt}$; proportional to $Q \to kQ$, inversely $\to \frac{k}{Q}$; in minus out.	Practice 1, 2, 5
Find k, then use the equation	Put one known amount and its rate in to find k; put the new amount in; say what it means.	E01, E02 · Practice 3, 4, 6

Example 01

Video · E01

A drug enters a patient's body through an IV at a constant rate of 4 mg per hour. The body removes the drug at a rate proportional to A, the amount of the drug in the body, in mg.

(a) Write a differential equation for $\frac{dA}{dt}$.

(b) When $A = 20$, A is increasing at 2 mg per hour. Find the constant of proportionality.

(c) Find $\frac{dA}{dt}$ when $A = 50$. What does it mean?

> Rate in minus rate out. Find k before you use $A = 50$.

Example 02

Video · E02

The rate of change of y with respect to x is proportional to the product of x and y. A solution curve passes through $(1, 2)$, where its slope is 6. For the same equation, what is $\frac{dy}{dx}$ at the point $(2, 3)$?

A. 6
B. 9
C. 12
D. 18

> Find k from the first point, then use it at the next.

VIDEO COMPLETE Check your example answers at the end of this booklet.

7.1 / Example answers

Modeling Situations with Differential Equations

E01 (a) $\frac{dA}{dt} = 4 - kA$; (b) $k = \frac{1}{10}$; (c) -1: when $A = 50$ mg, the amount is decreasing at 1 mg per hour

E02 (D)