Lecture Notes · Unit 6 · Lecture 11 AP CED 6.14

Selecting techniques for antidifferentiation

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INTRO: NO LABEL ON THE TEST

Which Method? AP Calculus 6.10 → 6.14

Title → u-sub

AP exam → no label

New integral → what first?

6.11-6.13 → BC only

AB → five forms, that's all

THE FIVE AB FORMS

AB antiderivatives: five forms

Step 0 → 6.8 basic rule? use it

Form 1 · composite → u = inside

→ $\int \cos x\, e^{\sin x}\,dx = e^{\sin x} + C$

Form 2 · top = (bottom)′ → u = bottom

→ $\int \frac{2x}{x^2+1}\,dx = \ln\left(x^2+1\right) + C$

Form 3 · top degree ≥ bottom → divide

→ $\frac{(x+1)^2}{x} = x + 2 + \frac{1}{x}$

Form 4 · $\frac{\ln x}{x}$ → u = ln x → $\int \frac{\ln x}{x}\,dx = \frac{(\ln x)^2}{2} + C$

5 · trig $\int \sec^2 x\,dx = \tan x + C$

No real roots → complete the square → arctan

Check → differentiate

THREE LOOK-ALIKES

Same bottom x^2+9, three tools

$\int \frac{x}{x^2+9}\,dx$ → top x, bottom′ 2x → Form 2

$u = x^2+9,\ x\,dx = \frac{1}{2}du \to \frac{1}{2}\ln\left(x^2+9\right) + C$

$\int \frac{1}{x^2+9}\,dx$ → Form 5 (6.10) → $\frac{1}{3}\arctan\frac{x}{3} + C$

$\int \frac{x^2}{x^2+9}\,dx$ → Form 3 → $\int \left(1 - \frac{9}{x^2+9}\right)dx$

$= x - 3\arctan\frac{x}{3} + C$

Same bottom → top first

PROBLEM 1 OF 3

Split the top, two tools

Find $\int \frac{2x+5}{x^2+2x+2}\,dx$.

Solution Name the form, then split the top

Plan: degrees 1 < 2 → Form 2: u = bottom

$u = x^2+2x+2 \to du = (2x+2)\,dx$

Top $2x+5 \neq 2x+2 \to 2x+5 = (2x+2)+3$

Split → $\int \frac{2x+2}{x^2+2x+2}\,dx + \int \frac{3}{x^2+2x+2}\,dx$

First → Form 2 → $\ln\left(x^2+2x+2\right)$

$x^2+2x+2 = (x+1)^2 + 1 > 0$ → no bars

$3\int \frac{dx}{(x+1)^2+1}$ → Form 5 → $3\arctan(x+1)$

$\ln\left(x^2+2x+2\right) + 3\arctan(x+1) + C$

Workbook → Problem 3

PROBLEM 2 OF 3

Which starts with long division?

Which integral is best started by long division?

(A) $\int \frac{x}{x^2+1}\,dx$

(B) $\int \frac{x^3}{x^2+1}\,dx$

(C) $\int \frac{1}{x^2+1}\,dx$

(D) $\int \frac{2x}{(x^2+1)^2}\,dx$

Solution Top first, then name each form

Plan: top first → name each form

(A) top x, bottom' 2x → Form 2, $u = x^2+1$

(C) top: a number → Form 5 → $\arctan x + C$

(D) Form 1 → $u = x^2+1$ → $\int u^{-2}\,du$

(B) $3 \geq 2$ → Form 3 → divide

So (B)

Workbook → 2, 4, 5

Judging by the bottom

✗ Common mistake

$\int \frac{x}{x^2+1}\,dx \to \arctan x$

Top of $(\arctan x)' \to 1 \neq x$

✓ Do this

x → Form 2

$u = x^2+1 \to \frac{1}{2}\ln(x^2+1)+C$

PROBLEM 3 OF 3

Rewrite, then u

Evaluate $\int_0^{\frac{\pi}{3}} \tan x\,dx$.

Solution Rewrite, then u

$(\tan x)' = \sec^2 x \to$ derivative

Plan: rewrite $\tan x = \frac{\sin x}{\cos x} \to$ Form 2

$u = \cos x \to du = -\sin x\,dx$

Bounds → $x = 0 \to u = 1$, $x = \frac{\pi}{3} \to u = \frac{1}{2}$

$\int_0^{\frac{\pi}{3}} \tan x\,dx = -\int_1^{\frac{1}{2}} \frac{1}{u}\,du$

$= \int_{\frac{1}{2}}^{1} \frac{1}{u}\,du \to$ bounds flipped

$= \left[\ln|u|\right]_{\frac{1}{2}}^{1} = \ln 1 - \ln\frac{1}{2}$

$= 0 - (-\ln 2) = \ln 2$

Workbook → 1, 6

WRAP-UP

Look at what first?

Basic rule? Else → name the form

Same bottom ≠ same tool → top first

Split the top · rewrite trig · differentiate to check

Next → Unit 7

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.