6.14 / Selecting Techniques for Antidifferentiation

Watch the lesson ➤

Ask in order: basic rule, rewrite, u, divide, complete the square; look at the top first

Learning goal

Pick the method for an integral by its form: 1) composite, $u =$ inside; 2) top $=$ (bottom)$'$, ln; 3) top degree $\geq$ bottom, divide; 4) $\frac{\ln x}{x}$, $u = \ln x$; 5) trig rules or complete the square. Look at the top first.

Key ideas

01 Which form is it? 1) A composite: $u =$ the inside. $\int \cos x\, e^{\sin x} dx$: $u = \sin x$, $du = \cos x\, dx$, $\int e^u\, du = e^{\sin x} + C$. 2) Top $=$ (bottom)$'$: $\int \frac{f'}{f} = \ln|f| + C$.

02 3) Top degree $\geq$ bottom: divide. $\frac{(x+1)^2}{x} = x + 2 + \frac{1}{x}$, so $\frac{x^2}{2} + 2x + \ln|x| + C$. 4) $\frac{\ln x}{x}$: $u = \ln x$, $du = \frac{1}{x}dx$, $\int u\, du = \frac{(\ln x)^2}{2} + C$. 5) Trig rules: $\int \sec^2 x\, dx = \tan x + C$; a constant over a bottom with no real zeros: complete the square, arctan.

03 Three look-alikes: $\frac{x}{x^2+9}$: top $= \frac{1}{2}$(bottom)$'$, so $\frac{1}{2}\ln\left(x^2+9\right) + C$. $\frac{1}{x^2+9}$: the basic rule with $a = 3$, $\frac{1}{3}\arctan\frac{x}{3} + C$. $\frac{x^2}{x^2+9}$: same degree, divide: $1 - \frac{9}{x^2+9}$, so $x - 3\arctan\frac{x}{3} + C$.

04 Look at the **top** first. Check by differentiating.

Remember

look-alikes

Integrals with the same bottom that need different tools; the top decides.

Class notes

6.14 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The wrong tool for a look-alike

✗ $\frac{x}{x^2+9} \to \arctan$; $\frac{x^2}{x^2+9} \to \ln$.

✓ Top = (bottom)′ (up to a constant): u = bottom. Same degree: divide.

6.14 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Pick the tool	Ask in order: basic rule, rewrite, u, divide, complete the square. Look at the top first.	E02, E03 · Practice 1, 2, 4, 5, 6, 7, 8
Two tools in one	Split the top: a multiple of the bottom's derivative (u = bottom), plus a constant (arctan).	E01 · Practice 3

Example 01

Video · E01

Find $\int \frac{2x+5}{x^2+2x+2}\,dx$.

> The derivative of the bottom is $2x + 2$. Split the top into $(2x + 2) + 3$.

Example 02

Video · E02

Which integral is best started by long division?

A. $\int \frac{x}{x^2+1}\,dx$

B. $\int \frac{x^3}{x^2+1}\,dx$

C. $\int \frac{1}{x^2+1}\,dx$

D. $\int \frac{2x}{(x^2+1)^2}\,dx$

> Compare the degree of the top with the degree of the bottom.

Example 03

Video · E03

Evaluate $\int_0^{\frac{\pi}{3}} \tan x \, dx$.

> Rewrite $\tan x = \frac{\sin x}{\cos x}$; the top is almost the derivative of the bottom.

VIDEO COMPLETE Check your example answers at the end of this booklet.

6.14 / Example answers

Selecting Techniques for Antidifferentiation

E01 $\ln\left(x^2+2x+2\right)+3\arctan(x+1)+C$

E02 (B)

E03 $\ln 2$