6.10 / Practice workbook

Integrating Using Long Division and Completing the Square

Practice

Problems 1–6 / Show your reasoning

1 Find $\int \frac{x+5}{x+2}\,dx$.

2 $\int \frac{1}{x^2+2x+2}\,dx =$

A. $\ln\left(x^2+2x+2\right)+C$

B. $\arctan(x+1)+C$

C. $\arctan(x+2)+C$

D. $\frac{1}{2}\arctan(x+1)+C$

3 Find $\int \frac{2x^2+x}{x-1}\,dx$.

4 $\int \frac{1}{x^2-6x+13}\,dx =$

A. $\frac{1}{2}\arctan\frac{x-3}{2}+C$

B. $\arctan\frac{x-3}{2}+C$

C. $\frac{1}{2}\arctan(x-3)+C$

D. $\ln\left(x^2-6x+13\right)+C$

5 Evaluate each integral.

(a) $\int_0^1 \frac{x^2+2}{x^2+1}\,dx$

(b) $\int_{-3}^0 \frac{1}{x^2+6x+18}\,dx$

6 $\int \frac{x^2+4x+6}{x+2}\,dx =$

A. $\frac{x^2}{2}+4x+6\ln|x+2|+C$

B. $x+2+\frac{2}{x+2}+C$

C. $\frac{x^2}{2}+2x+2\ln|x+2|+C$

D. $\frac{x^2}{2}+2x-\frac{2}{(x+2)^2}+C$

6.10 / Answer key

Integrating Using Long Division and Completing the Square

P01 $x+5=(x+2)+3 \;\rightarrow\; \dfrac{x+5}{x+2}=1+\dfrac{3}{x+2}$

$$\int\left(1+\frac{3}{x+2}\right)dx = x+3\ln|x+2|+C$$

P02 (B) $x^2+2x+2=(x+1)^2+1$

$u=x+1:\ \displaystyle\int\frac{du}{u^2+1}=\arctan(x+1)+C$

Answer B. (The top is not the derivative of the bottom, so not ln.)

P03 $2x^2+x-2x(x-1)=3x, \qquad 3x-3(x-1)=3$

$2x^2+x=(x-1)(2x+3)+3$

$$\frac{2x^2+x}{x-1}=2x+3+\frac{3}{x-1}$$

$$\int\left(2x+3+\frac{3}{x-1}\right)dx = x^2+3x+3\ln|x-1|+C$$

P04 (A) $x^2-6x+13=(x-3)^2+4, \quad a=2$

$u=x-3:\ \displaystyle\int\frac{du}{u^2+4}=\frac{1}{2}\arctan\frac{x-3}{2}+C$

Answer A.

P05 $1+\dfrac{1}{x^2+1} \;\rightarrow\; [x+\arctan x]_0^1=1+\dfrac{\pi}{4}$

$(x+3)^2+9,\ u=x+3:\ 0\rightarrow 3,\ \frac{1}{3}\left[\arctan\frac{u}{3}\right]_0^3=\frac{\pi}{12}$

P06 (C) $x^2+4x+6=(x+2)^2+2$

$$\frac{x^2+4x+6}{x+2}=x+2+\frac{2}{x+2}$$

$$\int\left(x+2+\frac{2}{x+2}\right)dx=\frac{x^2}{2}+2x+2\ln|x+2|+C$$

Answer C.