Lecture Notes · Unit 6 · Lecture 10 AP CED 6.10

Rewriting before integrating

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: STUCK ON A FRACTION

Rewrite, Then Integrate AP Calculus 6.10

$$\int \frac{x+3}{x+1}\,dx$$

No rule; top ≠ bottom' → no u. rewrite?

Divide · complete the square

THE RULES, ALL IN ONE PLACE

The rules we will use

$$x^2+bx+c=\left(x+\frac{b}{2}\right)^2+c-\frac{b^2}{4}$$

$$\int \frac{du}{u^2+1}=\arctan u+C$$

$$\int \frac{du}{u^2+a^2}=\frac{1}{a}\arctan\frac{u}{a}+C$$

$$\arcsin\frac{u}{a}+C$$

TOP-HEAVY: DIVIDE FIRST

Top-heavy → divide

$$\int \frac{x+3}{x+1}\,dx$$

degree top ≥ degree bottom → divide

Like $\frac{7}{2}=3+\frac{1}{2}$

$$x+3=(x+1)+2$$

$$\frac{x+3}{x+1}=1+\frac{2}{x+1}$$

$u=x+1$ $\int\left(1+\frac{2}{x+1}\right)dx=x+2\ln|x+1|+C$

QUADRATIC BOTTOM: COMPLETE THE SQUARE

Complete the square

$$\int \frac{1}{x^2+6x+10}\,dx$$

bottom' $2x+6\neq 1$ → no u

$36-40<0$ → no factoring

$$x^2+6x+10=(x^2+6x+9)+1=(x+3)^2+1$$

$u=x+3$, $du=dx$

$$\int \frac{du}{u^2+1}=\arctan(x+3)+C$$

WHY THE ONE OVER A

Why the $\frac{1}{a}$?

$$\int \frac{1}{x^2+4}\,dx$$

$$=\frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx$$

$u=\frac{x}{2}$, $du=\frac{1}{2}dx\rightarrow dx=2\,du$

$$\frac{1}{4}\cdot 2=\frac{1}{2}\rightarrow \frac{1}{2}\arctan\frac{x}{2}+C$$

$a=2\rightarrow \frac{1}{a}=\frac{1}{2}$

PROBLEM 1 OF 2

Divide, then integrate

Find $\int \frac{x^2+3}{x+1}\,dx$.

Solution Long division first

Plan: $2 \geq 1 \rightarrow$ divide

$x^2 + 3 = x^2 + 0x + 3$

$x \cdot (x+1) = x^2 + x$

$0x - x = -x + 3$

$-1 \cdot (x+1) = -x - 1, \quad 3 - (-1) = 4$

$$\frac{x^2+3}{x+1} = x - 1 + \frac{4}{x+1}$$

$$\rightarrow \frac{x^2}{2} - x + 4\ln|x+1| + C$$

Check $\rightarrow x - 1 + \frac{4}{x+1}$

Workbook → 1, 3, 6

Two ways to lose it

✗ Common mistake

$\ln|x+1|$? top ≠ bottom'

Drop $\frac{4}{x+1}$

✓ Do this

$$\frac{x^2+3}{x+1} = x - 1 + \frac{4}{x+1}$$

$\rightarrow 4\ln|x+1|$

PROBLEM 2 OF 2

Complete the square, then arctan

$\int_2^4 \frac{1}{x^2-4x+8}\,dx$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{8}$ (C) $\frac{\pi}{16}$ (D) $\frac{1}{2}\ln 2$

Solution Complete the square, then arctan

Plan: square → arctan

$16 - 32 < 0$

$$x^2 - 4x + 8 = (x-2)^2 + 4$$

$u = x - 2, \quad du = dx$

$2 \rightarrow 0, \; 4 \rightarrow 2$

$$\int_0^2 \frac{du}{u^2+2^2} = \frac{1}{2}\left[\arctan\frac{u}{2}\right]_0^2$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{8}$$

Solution Complete the square, then arctan

So (B); without $\frac{1}{2} \rightarrow$ (A)

2, 4, 5(b) → workbook

WRAP-UP

Stuck on a fraction?

Top-heavy → divide

Top = bottom' → u

No factoring → complete the square → arctan

Next → 6.14: choosing the method

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.