6.9 / Integrating Using Substitution

Watch the lesson ➤

The chain rule backward: u = the inside, du, the missing constant, and new ends

Learning goal

Integrate a composite by substitution: u = the inside first, du with the missing constant fixed, back to x, and new ends for a definite integral.

Key ideas

01 $\int 2x\cos\left(x^2\right)dx$: no basic rule fits. $u = x^2$, $du = 2x\,dx$: $\int \cos u\,du = \sin u + C = \sin\left(x^2\right) + C$. Check: $\left(\sin x^2\right)' = \cos\left(x^2\right)\cdot 2x$. This is **substitution**.

02 A missing constant: $\int xe^{x^2}dx$, $u = x^2$, $x\,dx = \frac{1}{2}du$: $\frac{1}{2}e^{x^2} + C$. $\int \cos 3x\,dx$: $dx = \frac{1}{3}du$: $\frac{1}{3}\sin 3x + C$.

03 Top = (bottom)′: $\int \frac{2x}{x^2+5}\,dx$, $u = x^2 + 5$: $\ln\left(x^2+5\right) + C$. In general $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

04 Change the ends: $u = x^2 + 1$; $x = 0 \to u = 1$, $x = 1 \to u = 2$: $\int_1^2 u^3 du = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$.

Remember

substitution

$u = g(x)$, $du = g'(x)\,dx$: $\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$.

Class notes

6.9 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Old ends, or no du

✗ Keep the old ends 1 and e after $u = \ln x$; or write du with no $\frac{1}{x}$.

✓ New ends $u = 0$ to 1, and $du = \frac{1}{x}dx$.

6.9 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Indefinite, by substitution	u = the inside, du = u' dx, match the constant, integrate in u, back to x.	E01, E03 · Practice 1, 2, 3, 7
Definite, new ends	Change the ends to u (or go back to x before using the old ends).	E02 · Practice 4, 5, 6

Example 01

Video · E01

$\int_1^e \frac{(\ln x)^2}{x}\,dx$ is

A. $\frac{1}{3}$

B. $\frac{e^3-1}{3}$

C. $\frac{e}{3}$

D. 1

Plan: $u = \ln x$; change the ends

Example 02

Video · E02

Find $\int \frac{e^x}{1+e^{2x}}\,dx$.

Plan: $e^{2x} = (e^x)^2 \rightarrow u = e^x$

Example 03

Video · E03

f is differentiable, and f' is continuous. The table gives values of f. Find $\int_0^2 f'(3x+1)\,dx$.

x	0	1	2	7
$f(x)$	3	5	6	-1

> Plan: $u =$ inside, new ends, $f' \to f$

VIDEO COMPLETE Check your example answers at the end of this booklet.

6.9 / Example answers

Integrating Using Substitution

E01 (A)

E02 $\arctan(e^x) + C$

E03 -2