Lecture Notes · Unit 6 · Lecture 8 AP CED 6.8

Antiderivatives and indefinite integrals

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INTRO: READ THE RULES BACKWARD?

Antiderivatives & Indefinite Integrals AP Calculus 6.8

Rate $f(x) \to F: F' = f$

Derivative rules, read backward?

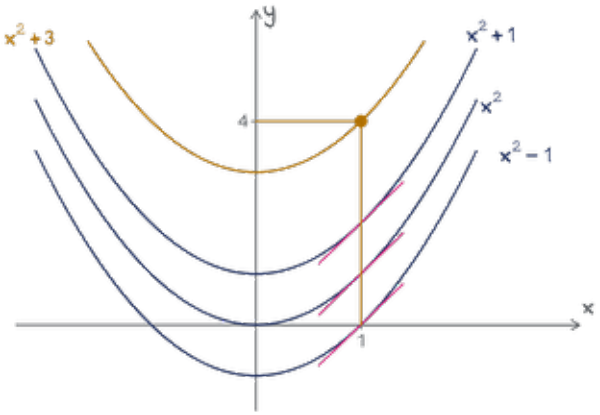
Backward → a whole family +C

$\frac{1}{x}$ breaks the pattern

THE FAMILY F(X) + C

Backward → a family

$\frac{d}{dx}x^2 = 2x$



$x^2 - 1, x^2, x^2 + 1 \to 2x$

At $x = 1$ → parallel, slope 2

$f(x) = 2x \to F(x) = ?$

$x^2 + 1, x^2 + 2, x^2 + 3, \ldots$

$F(x) = x^2 + C$, C any constant

$(F - G)' = 0 \to F - G = C$

Through $(1, 4) \to 1^2 + C = 4 \to C = 3$

1 point → $y = x^2 + 3$

THE INDEFINITE INTEGRAL AND ITS FOUR NAMES

The indefinite integral

No bounds → a family of functions

$\int 2x\,dx = x^2 + C$ → indefinite integral

$\int f(x)\,dx = F(x) + C$, where $F' = f$

F → antiderivative, C → constant of integration

f(x) → integrand, x → variable of integration

Indefinite → +C

INTEGRATE VS DIFFERENTIATE; NUMBER VS FUNCTION

Two orders, and number vs function

1. $\frac{d}{dx}\int f(x)\,dx = f(x)$

$2x \to x^2 + C \to 2x$

2. $\int \left(\frac{d}{dx}f(x)\right)dx = f(x) + C$

$x^2 \to 2x \to x^2 + C$

$\int_0^2 2x\,dx = 4, \int_0^2 2t\,dt = 4$

No bounds: $\int 2x\,dx = x^2 + C, \int 2t\,dt = t^2 + C$

Number vs function → $\int f(x)\,dx \neq \int f(t)\,dt$

THE POWER RULE, BACKWARD

$(x^3)' = 3x^{3-1} = 3x^2$

Backward → power +1, then ÷ new power

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

$\int x^{\frac{1}{2}}\,dx \rightarrow$ power $\frac{1}{2}+1 = \frac{3}{2}$

$\rightarrow \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}x^{\frac{3}{2}} + C$

Check → $\frac{2}{3}\cdot\frac{3}{2}x^{\frac{1}{2}} = x^{\frac{1}{2}}$ ✓

NEGATIVE POWERS, AND THE ONE REBEL: 1/X

Negative powers, and the rebel

$\int x^{-2}\,dx \rightarrow$ power $-2+1 = -1 \neq -3$

$\rightarrow \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$

$n = -1 \rightarrow \int x^{-1}\,dx \rightarrow \frac{x^0}{0}$ ✕ divide by 0

whose derivative is $\frac{1}{x}$?

$\int \frac{1}{x}\,dx = \ln|x| + C$

WHY THE BARS IN LN|X|

Why the bars?

$\ln x \rightarrow$ only $x > 0$; $\frac{1}{x} \rightarrow$ also $x < 0$

$x < 0 \rightarrow \frac{d}{dx}\ln(-x) = \frac{1}{-x}\cdot(-1) = \frac{1}{x}$

$\ln x$ right, $\ln(-x)$ left → $\ln|x|$ both

Never drop the bars; given $x > 0 \rightarrow \ln x$ OK

BASIC RULES: DERIVATIVES READ BACKWARD

Basic rules, read backward

Derivative, backward

$\int e^x\,dx = e^x + C;\ \int a^x\,dx = \frac{a^x}{\ln a} + C$

$\int \cos x\,dx = \sin x + C;\ \int \sin x\,dx = -\cos x + C$

$\int \sec^2 x\,dx = \tan x + C;\ \int \sec x\tan x\,dx = \sec x + C$

$\int \csc^2 x\,dx = -\cot x + C;\ \int \csc x\cot x\,dx = -\csc x + C$

$\int \frac{1}{1+x^2}\,dx = \arctan x + C;\ \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$

NO RULE FITS? REWRITE FIRST

$\int \frac{x^2+1}{\sqrt{x}}\,dx \rightarrow$ no quotient rule

Rewrite → $\frac{x^2}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}}} = x^{\frac{3}{2}} + x^{-\frac{1}{2}}$

Each term → $\frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + C$

$e^{-x^2} \rightarrow$ no elementary formula; → still one: $\int_0^x e^{-t^2}\,dt$

PROBLEM 1 OF 2

Divide first, then integrate

Find $\int \frac{4x^3-6x}{x^2}\,dx$.

Solution Divide first, then rule by rule

Plan: divide first → then rule by rule

$\frac{4x^3-6x}{x^2} = \frac{4x^3}{x^2} - \frac{6x}{x^2}$

$= 4x - \frac{6}{x}$

$-\frac{6}{x} = -6\cdot\frac{1}{x} \rightarrow n = -1 \rightarrow \ln|x|$

$$\int \left(4x - \frac{6}{x}\right)dx = 2x^2 - 6\ln|x| + C$$

$F'(x) = 4x - \frac{6}{x}$ ✓

Workbook → Problem 3, divide first

The fraction trap

X Common mistake

$\int \frac{4x^3 - 6x}{x^2}\,dx \neq$ top ÷ bottom

$\int \frac{6}{x}\,dx \neq \frac{6x^0}{0}$

No bars, no +C X

✓ Do this

Divide first → $4x - \frac{6}{x}$

$-\frac{6}{x} = -6 \cdot \frac{1}{x} \to -6\ln|x|$

$2x^2 - 6\ln|x| + C$

PROBLEM 2 OF 2

f from f' and one value

If $f'(x) = \frac{6}{\sqrt{1-x^2}} + 2x$ and $f(0) = 1$, then $f\left(\frac{1}{2}\right)$ is

(A) $\pi + \frac{1}{4}$ (B) $\pi + \frac{5}{4}$ (C) $\frac{\pi}{6} + \frac{5}{4}$ (D) $2\pi + \frac{5}{4}$

Solution Integrate with + C, then the point

3 steps: +C → C from (0, 1) → $f\left(\frac{1}{2}\right)$

$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$

$f(x) = 6\arcsin x + x^2 + C$

$f(0) = 6\arcsin 0 + 0^2 + C = 1$

$0 + 0 + C = 1 \to C = 1$

Solution Plug in one half

$f(x) = 6\arcsin x + x^2 + 1$

$\sin\theta = \frac{1}{2},\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\theta = \arcsin\frac{1}{2} = \frac{\pi}{6}$

$f\left(\frac{1}{2}\right) = 6 \cdot \frac{\pi}{6} + \frac{1}{4} + 1 = \pi + \frac{5}{4}$

(B) ✓ (A) $\pi + \frac{1}{4}$ → missing C

Given f'' → back twice → two constants, two facts

Workbook → Problems 4 to 6, one fact or two

WRAP-UP

Read the rules backward?

Backward → a family +C

No bounds → a function

$\frac{1}{x} \to \ln|x| + C$

$2x\cos(x^2) \to$ 6.9: undo the chain rule

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.