6.8 / Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation

Watch the lesson

Read derivative rules backward; + C; powers, 1/x, basic rules; rewrite first; one point fixes C

Learning goal

Find antiderivatives (indefinite integrals) with $+C$: powers backward, $\frac{1}{x} \to \ln|x|$, basic rules read backward, rewrite first, and fix C with one point.

Key ideas

01 $x^2 - 1$, x^2, $x^2 + 1$ all have derivative $2x$: same x, same slope. So $\int 2x\,dx = x^2 + C$, the **indefinite integral**: no ends, a family, not a number. Through $(1, 4)$: $1 + C = 4$, $C = 3$.

02 Powers backward: power +1, then divide. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$. $\int x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}} + C$; $\int x^{-2}\,dx = -\frac{1}{x} + C$.

03 $n = -1$ would divide by 0: $\int \frac{1}{x}\,dx = \ln|x| + C$ (for $x < 0$, $(\ln(-x))' = \frac{1}{x}$).

04 Basic rules backward: $\int e^x\,dx = e^x + C$, $\int \cos x\,dx = \sin x + C$, $\int \sin x\,dx = -\cos x + C$, $\int \sec^2 x\,dx = \tan x + C$, $\int \frac{1}{1+x^2}\,dx = \arctan x + C$, $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$.

05 Rewrite first: $\frac{x^2+1}{\sqrt{x}} = x^{\frac{3}{2}} + x^{-\frac{1}{2}}$, so $\int = \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + C$. Check by differentiating.

Remember

indefinite integral
$\int f(x)\,dx = F(x) + C$: every antiderivative of f.

constant of integration
The $+C$; one given point fixes it.

Class notes

6.8 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Integrating a quotient piece by piece, no + C

✗ $\int \frac{4x^3-6}{x^2}\,dx = \frac{\int(4x^3-6)\,dx}{\int x^2\,dx}$; or $2x^2 + \frac{6}{x}$ with no $+C$.

✓ Divide first, then term by term: $2x^2 + \frac{6}{x} + C$. Always $+C$.

6.8 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Antiderivatives by the rules	Rewrite as powers, then power + 1 and divide; 1/x -> ln|x|; basic rules backward; + C.	E01 · Practice 1, 2, 3
One point fixes C	Antidifferentiate with + C, then use the given value.	E02 · Practice 4, 5, 6

Example 01

Video · E01

Find $\int \frac{4x^3-6x}{x^2}\,dx$.

> Plan: not $\int \div \int \rightarrow$ divide, each term

Example 02

Video · E02

$f'(x) = \frac{6}{\sqrt{1-x^2}} + 2x$ and $f(0) = 1$. $f\left(\frac{1}{2}\right)$ is

A. $\pi + \frac{1}{4}$

B. $\pi + \frac{5}{4}$

C. $\frac{\pi}{6} + \frac{5}{4}$

D. $2\pi + \frac{5}{4}$

> Antidifferentiate with $+C$; then $f(0) = 1$ fixes C.

VIDEO COMPLETE Check your example answers at the end of this booklet.

6.8 / Example answers

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E01 $\dfrac{4x^3 - 6x}{x^2} = 4x - \dfrac{6}{x}$

$\dfrac{1}{x} = x^{-1}$: no power rule, it gives $\ln|x|$.

$$\int \left(4x - \frac{6}{x}\right) dx = 2x^2 - 6\ln|x| + C$$

Check: $\left(2x^2 - 6\ln|x|\right)' = 4x - \dfrac{6}{x}$.

E02 (B) $f(x) = 6\arcsin x + x^2 + C$

$f(0) = 0 + 0 + C = 1 \;\rightarrow\; C = 1$

$f\left(\dfrac{1}{2}\right) = 6 \cdot \dfrac{\pi}{6} + \dfrac{1}{4} + 1 = \pi + \dfrac{5}{4}$

Answer B.