6.7 / Practice workbook

The Fundamental Theorem of Calculus and Definite Integrals

Practice

Problems 1–7 / Show your reasoning

1 Evaluate $\int_1^3 (4x^3 - 2x)\,dx$.

2 $\int_0^2 (3x^2 + 2x - 1)\,dx =$

A. 8
B. 10
C. 15
D. 16

3 If $k > 0$ and $\int_0^k (2x + 1)\,dx = 12$, then $k =$

A. 2
B. 3
C. 4
D. 6

4 Let $f(x) = x^2 + 2x - 8$.

(a) Evaluate $\int_{-4}^2 f(x)\,dx$.

(b) Find the area of the region between the graph of f and the x-axis for $-4 \le x \le 2$.

5 A plant is 10 cm tall at week $t = 2$. Its height grows at a rate of $h'(t) = \frac{t}{2} + 1$ cm per week.

(a) Find the height at $t = 6$.

(b) Explain the meaning of $\int_2^6 h'(t)\,dt$ in context.

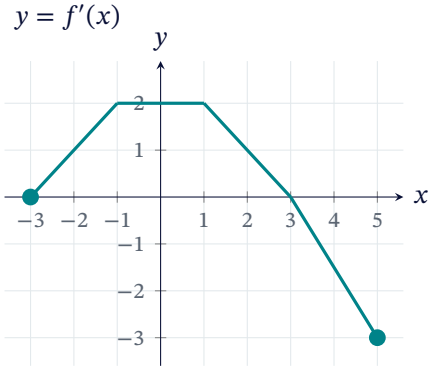
(c) At what time T is the plant 15 cm tall?

6 Let $f(x) = x^2$ for $x \leq 1$ and $f(x) = 2x - 1$ for $x > 1$. $\int_0^3 f(x)\,dx =$

A. 6
B. $\frac{19}{3}$
C. 7
D. $\frac{25}{3}$

7 The graph of f', made of segments, is shown for $-3 \leq x \leq 5$, and $f(1) = 4$.

$y = f'(x)$



(a) Find $f(5)$.

(b) Find $f(-3)$.

6.7 / Answer key

The Fundamental Theorem of Calculus and Definite Integrals

P01 $F(x) = x^4 - x^2$

$\left[x^4 - x^2\right]_1^3 = (81 - 9) - (1 - 1) = 72$

P02 (B) $F(x) = x^3 + x^2 - x$

$\left[x^3 + x^2 - x\right]_0^2 = 8 + 4 - 2 = 10$

Answer B.

P03 (B) $\int_0^k (2x + 1)\,dx = \left[x^2 + x\right]_0^k = k^2 + k$

$k^2 + k = 12 \rightarrow (k + 4)(k - 3) = 0$

$k > 0$, so $k = 3$. Answer B.

P04 $\left[\frac{x^3}{3} + x^2 - 8x\right]_{-4}^2 = -\frac{28}{3} - \frac{80}{3} = -36$

f = (x + 4)(x - 2) <= 0 on [-4, 2]: the whole region is below, area 36

P05 $h(6) = 10 + \int_2^6 \left(\frac{t}{2} + 1\right) dt = 10 + 12 = 22$

the plant grew 12 cm from t = 2 to t = 6 weeks

$10 + \left(\frac{T^2}{4} + T - 3\right) = 15 \rightarrow T^2 + 4T - 32 = 0 \rightarrow T = 4$

P06 (B) Split at $x = 1$, where the formula changes.

$\int_0^1 x^2\,dx + \int_1^3 (2x - 1)\,dx = \frac{1}{3} + \left[x^2 - x\right]_1^3 = \frac{1}{3} + 6 = \frac{19}{3}$

Answer B.

P07 $f(b) = f(1) + \int_1^b f'(x)\,dx$

$f(5) = 4 + \frac{1}{2} \cdot 2 \cdot 2 - \frac{1}{2} \cdot 2 \cdot 3 = 4 + 2 - 3 = 3$

$f(-3) = 4 - \int_{-3}^1 f'(x)\,dx = 4 - \left(\frac{1}{2} \cdot 2 \cdot 2 + 2 \cdot 2\right) = -2$

Going left over a positive area, f gets smaller.