Lecture Notes · Unit 6 · Lecture 7 AP CED 6.7

The Fundamental Theorem and definite integrals

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INTRO: NO SHAPE FORMULA

The Fundamental Theorem and Definite Integrals AP Calculus 6.7

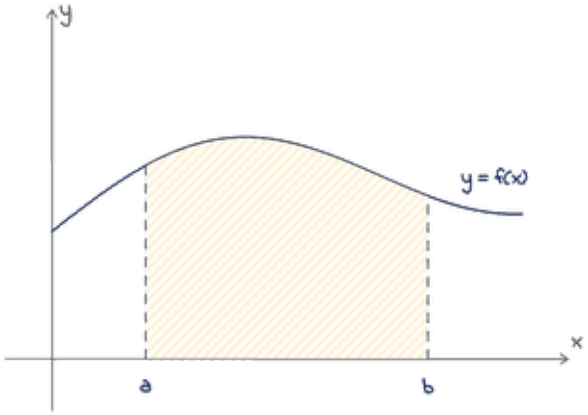
Area under $3x^2$ on $[1, 2]$ → no formula

Exact, in one line?

Same idea → start + change

F(B) – F(A): THE AREA IN ONE LINE

The area in one line



6.4 → $F' = f \rightarrow \int_a^x f(t)\,dt = F(x) - F(a)$

$\int_a^b f(x)\,dx = F(b) - F(a)$

F: its derivative is f → an antiderivative

6.4's area function → one antiderivative

$[F(x)]_a^b = F(b) - F(a)$ → top in, minus bottom in

f continuous on $[a, b]$ → FTC → 1F, 2 values

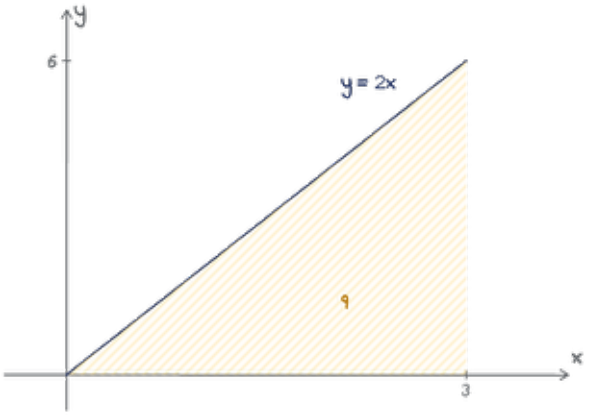
6.4: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

6.7: $\int_a^b f(x)\,dx = F(b) - F(a)$

2 directions

CHECK ON A TRIANGLE

Check it on a triangle



$\int_0^3 2x\,dx$ whose derivative is $2x$?

$(x^2)' = 2x \rightarrow F(x) = x^2$

$[x^2]_0^3 = 9 - 0 = 9$

Triangle: $\frac{1}{2} \cdot 3 \cdot 6 = 9$ ✓

$F = x^2 + 5$? → $(9 + 5) - (0 + 5) = 9$

+5 cancels → any antiderivative works

CURVES IN ONE LINE

$(x^3)' = 3x^2 \rightarrow$ antiderivative of $3x^2$: x^3

$\int_1^2 3x^2\,dx = [x^3]_1^2 = 8 - 1 = 7$

$\left(\frac{x^3}{3}\right)' = x^2 \rightarrow$ antiderivative of x^2: $\frac{x^3}{3}$

$(\sin x)' = \cos x$

$\int_0^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_0^{\frac{\pi}{2}}$

$= \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1$

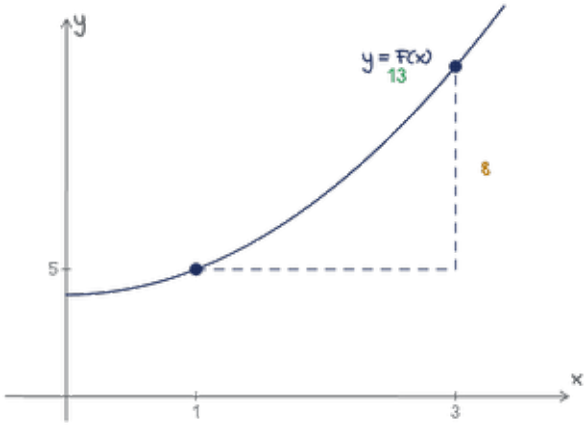
STARTING VALUE PLUS CHANGE

Starting value + change

$F(b) = F(a) + \int_a^b f(x)\,dx$

$= F(a) + (F(b) - F(a))$

$\rightarrow F(b) = F(b)$



$F(1) = 5,\ f(x) = 2x \rightarrow F(3)?$

$F(3) = 5 + \int_1^3 2x\,dx$

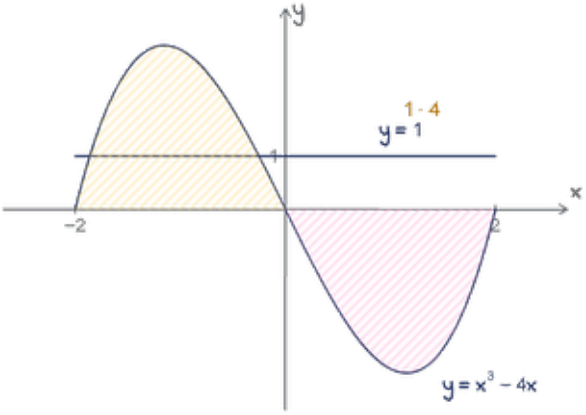
$\left[x^2\right]_1^3 = 9 - 1 = 8 \rightarrow 5 + 8 = 13$

AP: $f(b) = f(a) + \int_a^b f'(x)\,dx$

PROBLEM 1 OF 3

A symmetric interval

Evaluate $\int_{-2}^{2}(x^3 - 4x + 1)dx$. Use the graph of the integrand to explain your answer.



Solution Antiderivative, then the graph

Plan: $F \rightarrow F(2) - F(-2) \rightarrow$ graph

$F(x) = \frac{x^4}{4} - 2x^2 + x$

$F(2) = \frac{16}{4} - 8 + 2 = -2$

$F(-2) = 4 - 8 + (-2) = -6$

$-2 - (-6) = 4$

Graph: $x^3 - 4x$ is odd $\rightarrow A - A = 0$

1: even $\rightarrow 2\int_0^2 1\,dx = 2 \cdot 2 = 4$

$0 + 4 = 4$ ✓ $\rightarrow$ ends $-a$, a: odd part first

PROBLEM 2 OF 3

A starting value plus a change

If $f(2) = 7$ and $f'(x) = 3x^2 + 1$, then $f(4)$ is

(A) 58 (B) 65 (C) 68 (D) 75

Solution Start plus change

Plan: start + change

$$f(4) = f(2) + \int_2^4 (3x^2 + 1)\,dx$$

$$= 7 + \left[x^3 + x\right]_2^4$$

$$= 7 + (64 + 4) - (8 + 2) = 7 + 68 - 10$$

$$f(4) = 7 + 58 = 65$$

So (B)

Workbook : Problem 5

Don't drop the start

✗ Common mistake

58 → (A)

✓ Do this

$f(4) = 7 + 58 = 65$

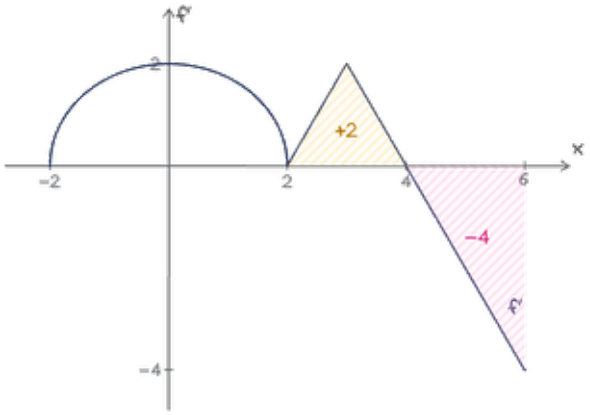
PROBLEM 3 OF 3

Values from the graph of f′

The graph of f' on $[-2, 6]$ has a semicircle and two segments, and $f(2) = 5$.

(a) Find $f(6)$.

(b) Find $f(-2)$.



Solution (a) Start plus change on a graph

Plan: start + change → change = signed area

(a)

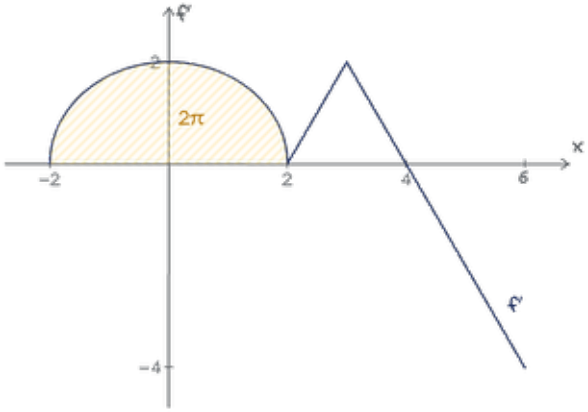
$$f(6) = f(2) + \int_2^6 f'(x)\,dx$$

$$\int_2^4 f'(x)\,dx = \frac{1}{2} \cdot 2 \cdot 2 = +2$$

$$\int_4^6 f'(x)\,dx = -\frac{2 \cdot 4}{2} = -4$$

$5 + 2 - 4 = f(6) = 3$

Solution (b) Left of the start



(b)

$f(-2)$: left of 2

$$f(-2) = f(2) + \int_2^{-2} f'(x)\,dx$$

$$= 5 - \int_{-2}^{2} f'(x)\,dx$$

$$A = \frac{1}{2}\pi(2)^2 = 2\pi$$

$$f(-2) = 5 - 2\pi$$

$f' > 0 \rightarrow f$ climbs to $x = 2 \rightarrow f(-2) < 5$ ✓

WRAP-UP



One line, and start + change

Yes: any antiderivative, top minus bottom

Start + change; left of the start → flip

e^x; $\frac{1}{x}$, trig → 6.8: every rule backward, +C

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.